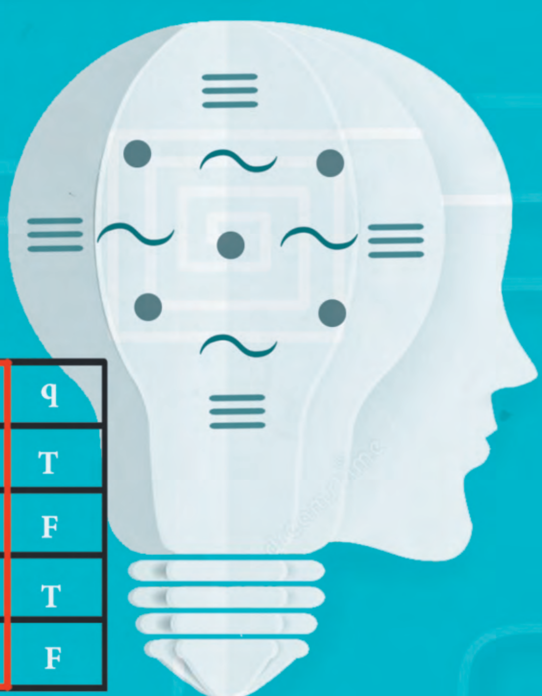
LOGIC

STANDARD XI



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The Constitution of India

Chapter IV A

Fundamental Duties

ARTICLE 51A

Fundamental Duties- It shall be the duty of every citizen of India–

(a) to abide by the Constitution and respect its ideals and institutions, the National Flag and the National Anthem;

(b) to cherish and follow the noble ideals which inspired our national struggle for freedom;

(c) to uphold and protect the sovereignty, unity and integrity of India;

(d) to defend the country and render national service when called upon to do so;

(e) to promote harmony and the spirit of common brotherhood amongst all the people of India transcending religious, linguistic and regional or sectional diversities, to renounce practices derogatory to the dignity of women;

(f) to value and preserve the rich heritage of our composite culture;

(g) to protect and improve the natural environment including forests, lakes, rivers and wild life and to have compassion for living creatures;

(h) to develop the scientific temper, humanism and the spirit of inquiry and reform;

(i) to safeguard public property and to abjure violence;

(j) to strive towards excellence in all spheres of individual and collective activity so that the nation constantly rises to higher levels of endeavour and achievement;

(k) who is a parent or guardian to provide opportunities for education to his child or, as the case may be, ward between the age of six and fourteen years.

The coordination committee formed by GR No Abhyas - 2116 (Pra. Kra. 43/16) SD-4 Dated 25.4.2016 has given approval to prescrie this textbook in its meeting held on 20.06.2019 and it has been decided to implement it from the educational year 2019-20.

LOGIC

STANDARD XI

2019

Maharashtra State Bureau of Textbook Production and Curriculum Research, Pune.

M3Y9E1

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The Constitution of India

Preamble

WE, THE PEOPLE OF INDIA, having solemnly resolved to constitute India into a SOVEREIGN SOCIALIST SECULAR DEMOCRATIC REPUBLIC and to secure to all its citizens:

JUSTICE, social, economic and political;

LIBERTY of thought, expression, belief, faith and worship;

EQUALITY of status and of opportunity; and to promote among them all

FRATERNITY assuring the dignity of the individual and the unity and integrity of the Nation;

IN OUR CONSTITUENT ASSEMBLY this twenty-sixth day of November, 1949, do HEREBY ADOPT, ENACT AND GIVE TO OURSELVES THIS CONSTITUTION.

NATIONAL ANTHEM

Jana-gana-mana-adhināyaka jaya hē
Bhārata-bhāgya-vidhātā,

Panjāba-Sindhu-Gujarāta-Marāthā
Drāvida-Utkala-Banga

Vindhya-Himāchala-Yamunā-Gangā
uchchala-jaladhi-taranga

Tava subha nāmē jāgē, tava subha āsisa māgē,
gāhē tava jaya-gāthā,

Jana-gana-mangala-dāyaka jaya hē
Bhārata-bhāgya-vidhātā,

Jaya hē, Jaya hē, Jaya hē,
Jaya jaya jaya, jaya hē.

PLEDGE

India is my country. All Indians are my brothers and sisters.

I love my country, and I am proud of its rich and varied heritage. I shall always strive to be worthy of it.

I shall give my parents, teachers and all elders respect, and treat everyone with courtesy.

To my country and my people, I pledge my devotion. In their well-being and prosperity alone lies my happiness.

PREFACE

Maharashtra State Bureau of Textbook Production & Curriculum Ressearch Pune, takes immense pleasure to introduce 'Logic' as a subject for Standrad XI. Logic is a science of reasoning. 'Rationality' is fundamental distinguishing characteristic of human being. This unique ability helps man to draw conclusions from the available information. Though ability to reason is an inbuilt feature of human beings, logicians have identified the rules of reasoning. Logic deals with these rules of reasoning. In logic one studies methods and principles of logic which enables one to distinguish between good and bad reasoning. Training in logic sharpens our ability to reason correctly and detect fallacies in reasoning if any. Logic, therefore is a fundamental discipline, useful to all branches of knowledge.

With the introduction of logic at Standard XI students will be able to understand, argue and convince with considerable amount of maturity. Study of logic will enrich their ability of logical, analytical and critical thinking.

The aim of this textbook is to explain basic principles of logic and their appllications. We have tried to make this textbook more interesting and activity based, which will facilitate easy understanding of the subject and create interest in the subject. The textbook is written keeping in mind needs of students from both urban and rural areas.

Various activity based questions, exercises, puzzles given in the textbook will help students to understand the basic concepts of logic and master the methods of logic. Q.R. code is given on the first page of the textbook. You will like the information provided by it.

The bureau of textbook is thankful to the Logic Subject Committee and Study Group, Scrutiny and Quality Reviewers and Artist for their dedication and co-operation in preparing this textbook.

Hope, Students, Teachers and Parents will welcome this textbook.

(Dr. Sunil Magar)
Director

Maharashtra State Bureau of
Text Book Production and
Curriculum Research, Pune

Pune
Date : 20 June 2019
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For Teachers

We are happy to introduce 'Logic' textbook for standard XI. As per the revised syllabus, two new topics are added in this textbook. These are : Origin and development of logic and Application of logic. Accordingly students will get brief information about historical development of Western as well as Indian logic. It will be interesting for students to know how logic has developed globally. Information about origin and development of Indian Logic will facilitate in enhancing pride in students mind about Indian contribution to the subject.

Logic is a fundamental subject and basis of all the branches of knowledge. The chapter, Application of logic illustrates the importance of logic in day to day life as well as in the important fields like - Law, Science and Computer science. This chapter will enable students to understand the importance of logical thinking while taking decisions in personal as well as professional life. They will also realize how taking rational decisions at right time can lead to success and happiness in life. Importance of logic can be highlighted by informing students, how study of logic can help them to appear for various competitive exams, which test the reasoning ability of students.

Logic as an independent subject is introduced as standard XI. At this stage students begin to think independently and express their thoughts and opinions. Logic being the science of reasoning, can help students in consistent and logical thinking. As teachers of logic it is our responsibility to train students to think rationally and reason correctly.

As Standard XI is the first year of stydying logic, it is necessary for teachers to take in to account students age and level of understanding. Logic studies abstract concepts, so the important concepts in logic need to be explained step by step, in easy to understand language and by giving examples and various activities in such a way that, students can relate the subject to their experiences in life. Keeping this in mind the textbook is made activity based. Teachers are expected to make use of various examples, teaching aids and activities like debates, logical puzzles and giving examples of good arguments and fallacies from everyday experience. In this way teaching and learning can become interesting and enjoyable experience for both students and teachers.

Competency Statements

Competency

- To acquire knowledge about the origin and development of logic.
- To understand the importance of logical thinking.
- To acquire knowledge about the fundamental concepts and principles of logic.
- To understand the types of argument and develop the ability to recognize the types of argument.
- To develop the ability of rational thinking.
- To understand the difference between sentence and proposition.
- To study the characteristics of propositions.
- To understand the types of propositions and to develop the ability to symbolize the propositions.
- To study the basic truth-tables.
- To study the method of truth-table.
- To develop the ability to apply the method of truth-table to decide whether a statement form is tautologous or not and to decide the validity of arguments.
- To study the method of deductive proof
- To develop the ability to prove the validity of deductive argument. by the method of direct deductive proof.
- To understand the need and importance of induction.
- To acquire knowledge about the types of inductive arguments and their use in our day to day life and science.
- To develop the ability of recognizing the types of inductive arguments.
- To enhance argumentation skills.
- To understand the different types of fallacies.
- To devleop the ability of recognizing the types of fallacies.
- To develop the ability to reason correctly and to detect errors in others argument.
- To understand the application of logic in day to day life, in the field of law, science and Computer science.

INDEX

Chapter 1 Nature of Logic

.... bad reasoning as well as good reasoning is possible, and this fact is the foundation of the practical side of logic. ---- CHARLES SANDERS PEIRCE

DO YOU KNOW THAT ....

Logic is a branch of philosophy.

Logic developed independently in India.

Ability to reason is the unique characteristic of man.

Logic will train you to reason correctly.

You need not have formal training in logic to use the rules of logic & reason correctly.

1.1 ORIGIN AND DEVELOPMENT OF LOGIC

Logic is traditionally classified as a branch of philosophy. Philosophy is fundamental to all spheres of human enquiry, and logic is the basis that strengthens philosophical thinking. In philosophy one needs to think clearly to deal with the most fundamental questions related to our life and this universe. Use of principles of logic in thinking, reasoning and arguments is central to the practice of philosophy.

In ancient times Logic originated and developed in India, Greece and China. The beginning of modern logic as a systematic study can be traced back to the Greek philosopher Aristotle (384-322 B.C.). Aristotle is regarded as the father of logic. The development of logic throughout the world is mainly influenced by the Aristotelian logic, except in India and China where it developed independently.

Logic originated in ancient India and continued to develop till early modern times. The Indian logic is represented by the Nyaya School of philosophy. The Nyaya Sutras of Akshapada Gautama (2nd century) constitute the core texts of the Nyaya School. In Mahabharata (12.173.45) and Arthashastra of Koutilya (Chanakya) we find reference of the Anviksiki and Tarka schools of logic in India. For his formulation of Sanskrit grammar, Panini (5th century BC) developed a form of logic which is similar to the modern Boolean logic.

The Buddhist and Jaina logic also comes under the Indian logic. Jain logic developed and flourished from 6th century BCE to 17th century CE. Buddhist logic flourished from about 500 CE up to 1300 CE. The main philosophers responsible for the development of Buddhist logic are Nagarjuna (c. 150-250 CE), Vasubandhu (400-800 CE), Dignaga (480-540 CE) and Dharmakirti (600-660 CE). The tradition of Buddhist logic is still alive in the Tibetan Buddhist tradition, where logic is an important part of the education of monks.

Mozi, "Master Mo", a contemporary of Confucius who founded the Mohist School, was mainly responsible for the development of logic in China. Unfortunately, due to the harsh rule of Legalism in the Qin Dynasty, this line of study in logic disappeared in China until Indian Logic was introduced by Buddhists.

Aristotelian logic is also known as traditional logic. Aristotle's logic reached its peak point in the mid-fourteenth century. The period between the fourteenth century and the beginning of the nineteenth century was largely one of decline and neglect. Logic was revived in the mid-nineteenth century.

At the beginning of a revolutionary period logic developed into a formal discipline. Logic is therefore classified as a formal science. The development of modern "symbolic" and "mathematical" logic during this period is the most significant development in the history of logic. As a formal science logic is closely related to the mathematics. Development in mathematics along with the contribution of thinkers like Leibniz, Francis Bacon, Augustus De Morgan, Bertrand Russell, George Boole, Peirce, Venn, Frege, Wittgenstein, Godel and Alfred Tarski has influenced the evolution of traditional logic in to today's modern logic.

Can you answer?

1. *If you attend lectures then you will understand the subject*
   You attend lectures
   Therefore ....
2. *Wherever there is smoke there is fire*
   There is smoke coming out from the building

Therefore ....

Solve the puzzles

1. *A famous mathematician was walking on a street. He saw a beautiful girl on a bus stop and he asked her, 'what is your name? The girl recognized him as a famous mathematician and replied that her name was hidden in the date 19/9/2001. Guess the girls name.*
2. *Manikchand was looking at the photo. Someone asked him, 'Whose picture are you looking at? He replied: "I don't have any brother or sister, but this man's father is my father's son. So whose picture was Manikchand looking at?*

1.2 DEFINITION OF LOGIC

We all can solve puzzles, give proofs and deduce consequences as illustrated above. This is possible because we are blessed with the ability to reason. This is the unique ability which differentiates man from other animals. This ability of ours is revealed when we infer, argue, debate or try to give proofs. We are born rational and may not require any formal training to reason. However our reasoning is not always good / correct / valid. Sometimes our reasoning is good and sometimes it is bad. It is necessary that we always reason correctly and this is where the role of logic is important because logic trains us to reason correctly.

Reason has applications in all spheres of human affairs. The study of logic, therefore, has applications in many important fields like Mathematics, Philosophy, Science, Law, Computer science, Education and also in our day to day life. Training in logic thus can help one in all the endeavors of life.

The word logic is derived from the Greek word 'Logos'. The word 'logos' means 'thought'. So **etymologically logic is often defined as, 'The science of the laws of thought.'** There are three types of sciences, 1) Natural sciences like physics, chemistry etc. 2) Social sciences like history, geography, sociology etc. and 3) Formal science like mathematics. Logic is a formal science. The etymological definition of logic, however, is not accurate, firstly because it is too wide and may lead to misunderstanding that logicians study the process of thinking, which is not correct. Thinking process is studied in psychology. Secondly the word 'thought' refers to many activities like remembering, imagining, day dreaming, reasoning etc. and logic is concerned with only one type of thinking i.e. reasoning.

Another very common and easy to understand definition of logic is – **'Logic is the science of reasoning.'** But this definition also is too wide. This definition restricts the study of logic only to reasoning but logicians are not interested in studying the process of reasoning as is implied by this definition too. **Logicians in fact are concerned with the correctness of the completed process of reasoning.**

The aim of logic is to train people to reason correctly and therefore the main task of logic is to distinguish between good reasoning and bad reasoning. This practical aspect of logic is accurately stated in I.M. Copi's definition of logic. He defines logic as – **'The study of the methods and principles used to distinguish good (correct) from bad (incorrect) reasoning.'** This definition is widely accepted by logicians.

Reasoning is a kind of thinking in which inference takes place i.e. a thinker passes from the evidence to the conclusion. The term **'inference'** refers to the mental process by which one proposition is established on the basis of one or more propositions accepted as the starting point of the process. **An argument** is a verbal representation of this process of inference and logic is mainly concerned with arguments. (In this text we shall use the words reasoning, inference and argument as synonyms)

1.3 SOME BASIC CONCEPTS OF LOGIC

To get precise understanding of the nature of logic it is further necessary to understand certain technical terms used in logic viz. 1) Argument 2) Valid argument 3) Form of argument. 4) True / False and Valid / Invalid.

1) Argument / Inference : An argument consists of proposition / statements. Every argument attempts to establish a proposition by giving another proposition / propositions in its support. **An argument may be defined as, 'A group of propositions in which one proposition is established on the evidence of remaining propositions.'** The proposition which is established is called the **conclusion** and the propositions which are stated in support of the conclusion are called **premises**. For instance in the given argument –

All artists are creative.
Sunita is an artist.
Therefore, Sunita is creative.

The propositions, 'All artists are creative' and 'Sunita is an artist' are premises and the proposition 'Therefore, Sunita is creative' is the conclusion which is established on the basis of evidence in the premises.

Thus premise (premises) and conclusion are the two basic constituent elements of an argument. In every argument the conclusion is derived from the premises and an attempt is made to show that the conclusion is a logical consequence of the premises.

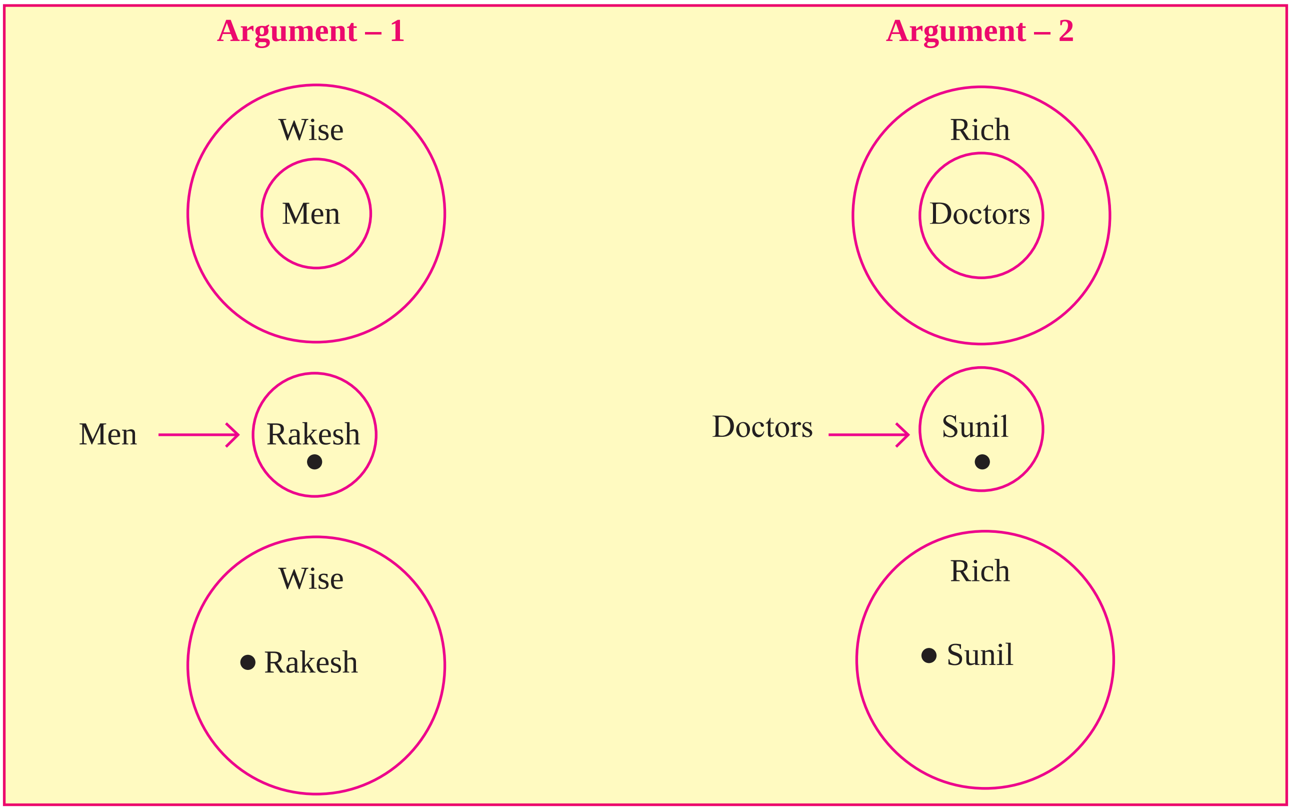
2) Valid argument : Every argument claims to provide evidence for its conclusion. However, every argument is not valid. The validity of an argument depends on the nature of relationship between its premises and conclusion. If the premises provide 'good' evidence for the conclusion, the argument is valid otherwise it is invalid. What is regarded as 'good' evidence, however, depends upon the type of argument.

3) Form of argument : The two important aspects of any argument are – form and content. Every argument is about something and that is the subject matter or the content of the argument. In the same way every argument has some form. Form means pattern or structure of the argument. For instance, pots may be of various shapes or patterns. These different shapes are the forms of pots. These pots may be made up of any material like clay, iron, bronze or silver. The material out of which it is made is the content of the pot. Now we may have pots of the same shape but made up of different material, we may have pots of the same material but of different forms or the pots differing in both form and matter. In the same way the arguments may differ in the content and have the same form, they may have the same content but different forms or they may differ both in the content and the form. For example –

(1) All men are wise.
Rakesh is a man.
Therefore, Rakesh is wise.

(2) All doctors are rich.
Sunil is a doctor.
Therefore, Sunil is rich.

The content or the subject matter of the above given arguments is different. The first argument is about men, wise and Rakesh. The second is about doctors, rich and Sunil. However, the form of both the arguments is same. The first premise of both the arguments states that a narrower class (men and doctors) is included in a wider class (wise and rich). The second premise of both the arguments states that an individual (Rakesh and Sunil) is a member of the narrower class. In the conclusion of both the argument it is inferred that the individual is, therefore, a member of the wider class. The following diagram clearly reveals how the form of both the arguments is same.



The form of the above arguments can also be expressed as follows ---

All A is B
X belongs to A
Therefore, X belongs to B

Can you give examples of ....

1. *Two arguments having different forms and same content?*
2. *Two arguments having different forms and different content?*

Can you state the form of the following arguments?

1. *All scientist are intelligent.*
   All intelligent are creative.
   Therefore, all scientists are creative.
2. *All men are rational.*
   Some rational beings are good.
   Therefore, some men are good.

4) True / False and Valid / Invalid

True / False and Valid / Invalid are important terms in logic. The terms valid / invalid are used for arguments in logic. **An argument is either valid or invalid and never true or false.** Validity of an argument depends upon the evidence in the premises for the conclusion. If the conclusion of an argument necessarily follows from the evidence in the premises then the argument is valid otherwise it is invalid.

An arguemnt consists of propositions / statements. **Proposition is either true or false. The terms valid / invalid are not used for propositions in logic.** A proposition is considered to be true if whatever is stated in the proposition agrees with actual facts, if not it is false. For example, 'Washington is an American city' is a true proposition. And 'London is an Indian city' is a false proposition.

1.4 DEDUCTIVE AND INDUCTIVE ARGUMENTS / INFERENCES

Can you find the difference in the evidence of following arguments?

1. If it rains then roads become wet.
It is raining.
Therefore, roads are wet.

2. All observed crows are black.
No observed crow is non-black.
Therefore, all crows are black.

Arguments are classified into two types 1) Deductive arguments 2) Inductive arguments. This classification of argumments into deductive and inductive is based on the nature of relationship between premises and conclusion. Premises of deductive arguments claim to provide sufficient evidence for the conclusion, whereas premises of inductive arguments provide some evidence for the conclusion.

Deductive Argument / Inference – Every argument attempts to prove the conclusion. The evidence needed to establish the conclusion is given in the premises. The evidence given in the premises is not always sufficient. **A deductive argument claims to provide conclusive grounds i.e. sufficient evidence for its conclusion.** If the **claim** that premises provide sufficient evidence is justified, the deductive argument is valid, if not it is invalid.

In a valid deductive argument where the evidence is sufficient the relation between the premises and the conclusion is of implication. Premises imply the conclusion means, if premises are true the conclusion is also true, it is impossible for the conclusion to be false. Thus the conclusion of a valid deductive argument is always certain.

Another important feature of a deductive argument is that, its conclusion is implicit in the premises i.e. the conclusion does not go beyond the evidence in the premises. This means we don't arrive at any new information. By deductive argument we can know what is implied by the premises. Deductive arguments do not give us any new information. For this inductive arguments are useful. Thus, the certainty of deductive arguments comes at a cost.

In an invalid deductive argument, however, the claim that premises provide sufficient evidence is not justified, therefore, the relation of implication does not hold between its premise and conclusion. Even when the premises are true the conclusion may be false. For example, let us consider the following arguments.

(1) If Amit passes S.S.C. with good marks, he will get admission in college.
Amit passed S.S.C. with good marks.
Therefore, he well get admission in college.

(2) Meena will either go to college or study at home.
Meena did not go to college.
Therefore, Meena is studying at home.

(3) If Anita gets the prize then she will become famous.
Anita did not get the prize.
Therefore, Anita will not become famous.

(4) If it rains heavyly, the college will declare holiday.
College has declared a holiday.
Therefore, it is raining heavily.

All these arguments are deductive arguments as the conclusions of all the arguments don't go beyond the evidence in the premises. The first two arguments are valid as premises provide sufficient evidence. The premises imply the conclusion. If premise are true, conclusion cannot be false. The last two arguments, though deductive, are not valid because the claim that premises provide sufficient evidence is not justified. Even when premises are true, the conclusion may be false. So there is no relation of implication, the conclusion does not necessarily follow from the premises.

The deductive arguments are formally valid. **A formally valid argument is one whose validity is completely determined by its form.** In case of deductive arguments the content of its premises and conclusion does not affect its validity. There is no need to judge the content of the premises and conclusion, also there is no need to find out whether they are true or false to determine the validity. One only needs to check the form of the argument. If the form is valid the argument is also valid. For example –

(1) All men are animals.
All animals are mortals.
Therefore, all men are mortals.

(2) All crows are birds.
All birds have wings.
Therefore, all crows have wings.

(3) All singers are actors.
All actors are leaders.
Therefore, all singers are leaders.

(4) All cats are rats.
All rats are lazy.
Therefore, all cats are lazy.

The form of all the above given deductive arguments is as follows :

All X is Y.
All Y is Z.
Therefore, All X is Z.

It is obvious that the form is valid and therefore all the arguments being its substitution instances are also valid. It is easy to accept that the first two arguments are valid because the premises and conclusions of these arguments are all true and conclusion necessarily follows from the premises. But one may find it difficult to accept that, the third and fourth argument is valid as premises and conclusion of both the arguments are false. However they are also valid. Validity of deductive argument is conditional. **In case of a valid deductive argument if premises are true the conclusion must be true.** So if premises of the last two arguments are assumed as true then the conclusions of both the arguments necessarily follow from the premises and therefore both the arguments are valid. If conclusion necessarily follows from the premises then the deductive argument is valid. Premises and conclusion of valid deductive argument may or may not be true. **When the deductive argument is valid and its premises and conclusion are true, such an argument is called sound argument.**

As deductive arguments are formally valid, the validity of deductive arguments can be determined or proved by using the rules and methods developed by logicians.

Inductive Argument / inference --- Inductive argument is an argument which provides some evidence for the conclusion. The conclusion of an inductive argument goes beyond the evidence in the premises. There is a guess, prediction or something new is asserted in the conclusion for which the evidence given in the premises is not sufficient. As the evidence in the premises is not sufficient, the premises of an inductive argument don't imply the conclusion. This means even when the premises are true the conclusion may be false. The conclusion of an inductive argument is always probable. Whether the argument is good (valid) or bad (invalid), the possibility of its conclusion being false always remains.

Technically the terms 'valid' and 'invalid' cannot be used for inductive arguments. Only deductive arguments are either valid or invalid. Inductive arguments can be judged as better or worse. More the possibility of the conclusion being true, better the argument. The addition of new premises may alter the strength of an inductive argument, but a deductive argument, if valid, cannot be made more valid or invalid by the addition of any premises. We shall use the terms 'good' or 'bad' for inductive arguments. For example, consider the following arguments.

(1) Whenever cat crossed my way in the past, something bad happened on that day.
Today morning a cat crossed my way.
Therefore, I am sure that something bad is going to happen today.

(2) Every morning I have seen the sun rising in the east.
It is early morning now.
So, I am sure I will find sun rising in the east.

(3) The doctor told me that, Suresh is suffering from cancer and he will not survive for more than three months.
After two months I got the news that Suresh is no more.
So, Suresh must have died due to cancer.

All the above given arguments are inductive arguments as conclusions of all the arguments go beyond the evidence in the premises. The premises don't imply the conclusion. Even if premises are true the conclusions of all the arguments are probable. The conclusion is probable does not mean that the argument is bad. In the above given arguments the first one is bad where as the other two are good.

Like deductive arguments the validity of inductive arguments i.e. whether the inductive argument is good or bad, is not determined by the form of the argument, but is decided by its content. Inductive arguments are materially valid. **A materially valid inference is one whose validity is completely determined by its content.** To decide whether the given inductive argument is good or bad, one has to consider the content / the subject matter of the argument. The form of the first and second argument is the same but the first one is bad whereas the second one is good.

The amount of evidence in the premises determines whether the argument is good. If the evidence in the premises makes it reasonable to accept the conclusion, then, the argument is good otherwise it is bad. From the above given arguments, the first arguments is a bad one because the conclusion is based on the superstition, there is no connection between a cat crossing the way and good or bad events happening in our life. In the other two arguments, though, the conclusions may turn out to be false, the evidence on the basis of which the conclusions are derived is scientific. Hence the last two arguments are good.

Though the content decides whether an inductive argument is good, this does not mean that the premises and conclusion of good inductive arguments are true and of bad inductive arguments are false. In case of the first argument, even if premises are true and the conclusion turns out to be true, still the argument is bad. Similarly in case of the last argument even if conclusion turns out to be false when the premises are true, the argument is good because the inference is based on the doctor's verdict.

Like deductive arguments, whether the given inductive argument is good or bad cannot be determined by the methods and rules of logic. In case of common man's inductive arguments, as given above, one can easily decide whether they are good or bad. However, in case of the inductive arguments, in various sciences, by judging the evidence in the premises only the experts in the field can decide whether it is good or bad. Unlike deductive arguments, the Inductive arguments, provide us with new information and thus may expand our knowledge about the world. So, while deductive arguments are used mostly in mathematics, most other fields of research make extensive use of inductive arguments.

Truth and Validity of arguments – The relation between validity or invalidity of the argument and truth or falsity of its premises and conclusion is not simple. As discussed earlier, an argument may be valid when one or more or even all its premises and conclusion are false and an argument may be invalid with all its premises and conclusion true. **The truth or falsity of an argument's conclusion does not by itself determine the validity or invalidity of that argument. And the fact that an argument is valid does not guarantee the truth of its conclusion.**

DEDUCTIVE ARGUMENT	INDUCTIVE ARGUMENT
1. Premises claim to provide sufficient evidence for the conclusion.	1. Premises provide some evidence for the conclusion.
2. In valid deductive argument premises imply the conclusion.	2. Premises do not imply the conclusion.
3. In valid deductive argument if premises are true, conclusion must be true.	3. Even when premises are true conclusion may be false.
4. Conclusion of valid deductive argument is always certain.	4. Conclusion is always probable.
5. Conclusion does not go beyond the evidence in the premises.	5. Conclusion goes beyond the evidence in the premises.
6. Arguments are formally valid.	6. Arguments are materially valid.
7. Validity can be determined by rules and methods of logic.	7. Correctness of arguments can be decided by an appeal to experience and not by rules and methods of logic.
8. Deductive arguments cannot expand our knowledge of the world, by deduction we can only know what is implied by the premises.	8. With inductive arguments we can discover something new and expand our knowledge of the world.

Summary

- In past logic developed independently in India, Greece and China.
- Modern logic is evolved from Aristotelian or traditional logic.
- Logic is study of methods and principles used to distinguish between good and bad reasoning.
- Arguments, Valid argument, Form of argument, True / False, Valid / Invalid are some important concepts in logic.
- The two important types of arguments are – Deductive and Inductive arguments.
- Deductive arguments claim to provide sufficient evidence for the conclusion.
- Inductive arguments provide some evidence for the conclusion.

Exercises

Q. 1. Fill in the blanks with suitable words given in the brackets.

1. .... is regarded as the father of logic. *(Aristotle / De Morgan)*
2. The development of logic throughout the world is mainly influenced by the .... logic. *(Aristotelian / Indian)*
3. The Nyaya Sutra of .... constitute the core texts of the Nyaya School. *(Gautama / Nagarjun)*
4. The proposition which is established in the argument is called the .... *(Conclusion / Statement)*
5. The proposition which is stated in support of the conclusion is called .... *(Premise / Conclusion)*
6. .... means pattern or structure of the argument. *(Content / Form)*
7. .... is either valid or invalid. *(Proposition / Argument)*
8. A deductive argument claims to provide .... evidence for its conclusion. *(Some / Sufficient)*
9. In Inductive argument premises provide .... evidence for the Conclusion. *(Some / Sufficient)*
10. In case of a valid .... argument if premises are true the conclusion must be true. *(Deductive / Inductive)*
11. A materially valid inference is one whose validity is completely determined by its .... *(Content / Form)*
12. Conclusion of valid deductive argument is always .... *(Certain / Probable)*
13. Validity of .... arguments can be determined by rules and methods of logic. *(Deductive / Inductive)*
14. Correctness of .... arguments is determined by an appeal to experience. *(Deductive / Inductive)*
15. Conclusion of .... inference does not go beyond the evidence in the premises. *(Deductive / Inductive)*

Q. 2. State whether following statements are true of false.

1. Logic is a branch of Psychology.
2. Philosophy is fundamental to all spheres of human enquiry.
3. The Jaina logic is represented by the Nyaya School of philosophy.
4. Mozi, "Master Mo" was mainly responsible for the development of logic in China.
5. Etymologically logic is often defined as the science of the laws of thought.
6. Form means pattern or structure of the argument.
7. Argument is either true or false.
8. The classification of arguments into deductive and inductive is based on the nature of relationship between premises and conclusion.
9. When the deductive argument is valid and its premises and conclusion are true, such an argument is called sound argument.
10. A formally valid argument is one whose validity is completely determined by its content.
11. Conclusion of inductive is always certain.
12. Conclusion of inductive argument goes beyond the evidence in the premises.
13. Even when premises are true conclusion of valid deductive argument may be false.
14. The truth or falsity of an argument's conclusion does not by itself determine the validity or invalidity of that argument.
15. Deductive arguments cannot expand our knowledge of the world.

Q. 3. Match the columns.

	(A)		(B)
1.	Nyaya	1.	Sufficient evidence
2.	Aristotle	2.	Mozi ‘Master Mo’
3.	Mohist school	3.	Some evidence
4.	Nagarjun	4.	Valid or Invalid
5.	Argument	5.	Greek logician
6.	Statement	6.	Buddhist philosoper
7.	Deductive argument	7.	Akshapad Gautama
8.	Inductive argument	8.	True or false

Q. 4. Give logical terms for the following :

1. The study of methods and principles used to distinguish good from bad resoning.
2. A proposition that is stated in support of the conclusion in an argument.
3. The proposition that is established in the argument.
4. An argument that claims to provide sufficient evidence for its conclusion.
5. An argument in which premises provide some evidence for the conclusion.
6. An argument whose validity is completely determined by its form.
7. An argument whose validity is completely determined by its content.

Q. 5. Give reasons for the following :

1. Etymological definition of logic is not accurate.
2. Deductive arguments cannot expand our knowledge of the world.
3. Conclusion of valid deductive argument is always certain.
4. Conclusion of an inductive argument is always probable.

Q. 6. Explain the following.

1. Truth and validity.
2. Form of argument.
3. Distinction between form and content.
4. Distinction between formal and material validity.
5. Distinction between deductive and inductive argument.

Q. 7. Answer the following questions.

1. Explain in brief origin and development of logic.
2. Write short note on Indian Logic.
3. Define logic and explain the terms - Argument, Premise and Conclusion.
4. Explain the difference between terms - Reasoning, Inference and Argument.
5. Explain with illustration nature of Deductive argument.
6. Explain with illustration nature of Inductive argument.

Q. 8. State whether the following arguments are deductive or Inductive.

1. Either it is a bank holiday or the bank is open. It is not a bank holiday. Therefore the bank is open.
2. There are no good players in our college team. So the team will not win the match.
3. Whenever I went to my sister’s house she cooked biryani for me. As I am visiting my sister today, I am sure my sister will make biryani.
4. My aunty is a doctor, so she is a female doctor.
5. If Mohan takes admission for science then he will take computer science. Mohan has taken admission for science. So he must have opted for computer science.
6. Meena is smart. Seema is smart, Neena is smart. These are all girls. Therefore all girls are smart.

7. Sunil is hardworking, intelligent and smart. Therefore Sunil is smart.

8. Nikita is not happy with her job, so I am sure she will leave the job.

9. Mukesh is an actor and Mukesh is handsome. Therefore Mukesh is handsome actor.

10. If I go to college then I will attend lecture. If I attend lecture then I will understand logic and if I understand then I will pass with good marks. Therefore if I go to college then I will pass with good marks.

11. Amit and Sumit are in same class, they both play cricket and go to same tuition class. Amit is a good singer. Therefore Sumit is also a good singer.

12. India has taken loan from the world bank, so India is sure to develop economically.

13. If and only if a student is sick during examination, he is allowed to appear for re-examination. Ashok is allowed to appear for re-examination. So Ashok must have been sick during examination.

14. Suresh is taller than Naresh. Naresh is taller than Ramesh. Therefore Suresh is taller than Ramesh.

15. Hardly any man lives for more than hundred years. Mr. Joshi is ninety nine year old. So he will die next year.

❖ ❖ ❖

Chapter 2 Nature of Proposition

Logic studies the preservation of truth and propositions are the bearers of truth and falsity.

Identify the following arguments.

EXAMPLE 1.	*EXAMPLE 2.*
All men are mortal.	All actors are handsome.
All artists are men.	Prasad is an actor.
Therefore, all artists are mortal.	Therefore, ---------------

We have seen earlier that one of the functions of logic is to study arguments. However, to study the arguments, it is essential to understand the statements that constitute an argument.

We begin by examining propositions, the building blocks of every argument. An argument consists of premises and conclusion. These **premises** and **conclusion** are in the form of **propositions or statements**. Hence, a proposition is a basic unit of logic.

Find the premises and the conclusion from the following :

EXAMPLE 1

All monuments are beautiful.

Taj Mahal is a monument.

Therefore, Taj Mahal is beautiful.

EXAMPLE 2

All Mangoes are fruits

All fruits grows on trees.

Therefore, All mangoes grow on trees.

2.1 PROPOSITION (STATEMENT) AND SENTENCE

Definition of proposition –

A proposition is defined as a sentence, which is either true or false.

Activity : 1

Make a list of true or false propositions.

From the definition of proposition we can conclude that **all propositions are sentences but all sentences are not propositions**. Only those sentences which are either true or false will be propositions. Hence, the class of proposition is narrow, whereas the class of sentences is wider. This leads to a question that, which sentence can be true or false? To answer this question we shall have to consider various kinds of sentences.

Activity : 2

Make a list of sentences you know and state its kind.

Kinds of Sentences :

(1) Interrogative Sentence : This kind of sentence contains a question.

Example : What is your name?

(2) Exclamatory Sentence : It is a kind of sentence which expresses some kind of feelings.

Example : Oh! God

(3) Imperative Sentence : This sentence expresses a command or an order.

Example : Get Out.

(4) Optative Sentence : This sentence expresses a wish, desires, urges.

Example : May God bless us all.

(5) Assertive Sentence : It is a sentence which asserts something about an individual. This sentences can make positive or negative assertion. (It refers to identifiable particular individual possessing definite, particular property.)

The word "Individual" stands not only for persons but for anything like city, country, animal, or anything to which attributes can be significantly predicated and the property/ attribute may be an adjective, noun, or even a verb.

Example : Sanika visits her grandmother during the holidays. (Positive assertion)

Example : The tiger is not a domestic animal. (Negative assertion)

These kind of sentences can be either true or false. Hence, they are considered as statements or propositions in logic. They are also called as declarative sentences. They are informative sentences because they provide us with information. So declarative sentences can make logical propositions. So, we can conclude that all sentences are not propositions. Only those sentences which can be either true or false can be propositions.

Sometimes declarative sentence may be in the form of a question or an exclamation e g. (1) Do you feel you can fool your friend.

(2) Thief .... Thief

Grammatically given examples are Inerrogative and exclamatory sentences respectively but logically they are propositions .

Activity : 3

Make a list of Assertive / Declarative / Informative sentences.

PICTURE: 1.

PICTURE: 2

Activity : 4

Observe and describe these pictures and make a list of assertive propositions.

(positive assertion and negative assertion)

A proposition is expressed in the form of a sentence. But it is not the same as sentence. The same proposition may be expressed by different sentences.

Example : (1) This is a fish (English)

(2) Das ist ein fisch (German)

(3) यह मछली है। (Hindi)

(4) हा मासा आहे. (Marathi)

(5) kore wa sakana desu. (Japanese)

Here a sentence in English, Marathi, Hindi, German, Japanese may differ as sentence but they express the same proposition.

Anything that is known through sense organs has physical existence. A proposition refers to the meaning or content expressed in the form of a sentence. Therefore, it does not have a physical existence. It is expressed through the medium of a sentence.

On the other hand a sentence has a physical existence. A sentence when spoken, is in the form of sound waves. When written, it is a sign or a symbol on a surface. e.g. In five different sentences given above. The meaning expressed in these sentences is the proposition which does not have a physical existence because one cannot see it, touch it but one can understand it if and only if the language in which it is expressed is known.

The following are the **main characteristics of proposition :**

(1) Every proposition has a truth value :

The truth or falsity of proposition are called truth values. The truth value of a true proposition is true and that of a false proposition is false.

Now the question arises, "what determines the truth value of a proposition?

The answer is "The Fact".

If a proposition represents a fact or facts, it is true. It means a proposition is true when the assertion in a proposition {that which is said in a proposition} agrees with the facts.

Example : Butter melts in heat.

If a proposition does not represents a fact and if the claim is not justified then, the proposition is false.

Example : Mumbai is capital of India. (truth value of this proposition is false)

(2) A proposition has only one truth value.

A proposition cannot be true and false together.

E. g. Chalk is white. (This proposition cannot be both true and false.)

(3) The truth value of a proposition is definite :

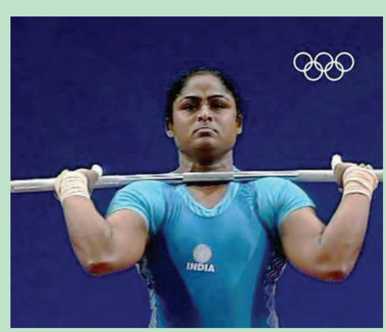
A proposition has unique truth value. If a proposition is true, it is always true. If it is false, it is always false. In other words truth value of a proposition does not change.

Example : The earth is a flat disc.

Though, the truth value of the above proposition appears to have changed but in reality it not so. This proposition was believed to be true due to ignorance (lack of scientific knowledge) but it is proved to be false today.

Thus, all propositions are sentences but all sentences are not propositions. Only those sentences which are either true or false are propositions.

Activity : 5 Look at the pictures carefully and construct the propositions describing the pictures.



There are important differences between the proposition and sentence. Yet they are interconnected.

Proposition (Statement)	Sentence
(1) It is sentence which is either true or false.	(1) It is a meaningful group of words in a grammatical order.
(2) A proposition is conveyed through a sentence.	(2) A sentence is a vehical through which a statement is expressed.
(3) Only declarative sentences are proposition.	(3) The sentences which expresses feeling, wish etc are sentences only.
(4) Every proposition has a truth value i.e it is either true or false.	(4) Sentence does not have a truth value. It is neither true nor false.
(5) A proposition does not have physical existence.	(5) A sentence has a physical existence.
(6) ***Example :*** Taj Mahal is white.	(6) ***Example :*** How are you?

2.2. Classification of proposition :

Classification of proposition can be done on the basics of whether the statement contains another statement as it's component propositions. Some propositions do not contain another proposition as a component, while others do. The former are called simple proposition and the later are compound propositions.

Simple Proposition :

It is a basic unit in logic. **Simple proposition is defined as a proposition that does not contain any other proposition / propositions as it's component.**

Example :

(1) Delhi is the capital of India.

(2) Peacock generally live in jungle.

(3) Polygon has six sides.

(4) Turmeric reduces my arthritis pain.

(5) Anil is eligible to drive.

(6) Mumbai is the capital of England

Activity : 6

Make a list of simple propositions.

Compound proposition :

Example :

(1) (Delhi is the capital of India) **and** (it is a crowded city).

(2) (Peacock generally lives in jungle) **or** (bushes.)

(3) **If** (polygon has six sides) **then** (it is a hexagon.)

(4) **If** (turmeric reduces my arthritis pain) **then** (I will eat turmeric everyday).

(5) (Anil is eligible to drive) **if and only** if (he is eighteen years old).

(6) **It is false** that (Mumbai is the capital of England).

Proposition that comes as a part of a proposition is called as **a component proposition.** The propositions in a compound statements are called its components.

Activity : 7

Identify Component proposition from the above example.

Thus, **a compound proposition is defined as a proposition which contains another proposition / propositions as its component.**

Activity : 8

Construct compound propositions using statements constructed by you in **Activity 6**.

Kinds of simple proposition :

There are four kinds of simple proposition. These are :

(1) Subjectless proposition :

The simplest kind of proposition is the subjectless proposition.

Example :

(1) Bomb!

(2) Fire!

Subjectless propositions make an assertion. They give information. Therefore they are propositions. However the subject of the assertion is not clear. They are primitive propositions.

(2) Subject – Predicate proposition :

A subject – predicate proposition states that an individual possesses a quality or attribute. A subject predicate proposition is that which has a subject , a predicate and a verb. An individual is a singular term. Therefore, **the subject of this kind of proposition is a singular term**.

Example : Ashok is intelligent.

(3) Relational Proposition :

A relational proposition states a relation between two subjects. The subjects between which a relation is stated are called **terms of relation.**

Example : Ram is taller than Shyam.

The above proposition expresses a relation between two subjects namely Ram and Shyam.

(4) Class membership proposition :

A class membership proposition asserts that an individual is a member of a class. Thus, it shows that the subject term belongs to the class indicated by predicate. So, here **predicate term is general.**

Example :

(1) Rani Lakshmi bai was a great warrior.

(2) Bhagat Singh was a freedom fighter.

Kinds of compound proposition :

Compound proposition are further classified into two kinds –

(1) Truth – functional compound proposition

(2) Non Truth – functional compound proposition.

(1) Truth functional compound proposition:

In a compound proposition there are two or more component propositions that are connected by some expression like ‘and’, ‘or’ etc. These component propositions are either true or false. The component proposition as a whole also has some truth value.

Example : Sameer is intelligent and Sameer is smart.

In this proposition there are two propositions

(1) Sameer is intelligent.

(2) Sameer is smart.

Now when there are two component propositions, we get four possibilities as given below :

Sameer is intelligent	**And**	He is smart
TRUE	TRUE	TRUE
TRUE	FALSE	FALSE
FALSE	FALSE	TRUE
FALSE	FALSE	FALSE

The truth value of compound propositions (which is stated in the middle column) changes as per the truth value of its component proposition.

In the above example when both the components are true one can say that the compound proposition is true. Otherwise under other possibilities it is false.

Thus the **truth functional compound proposition is defined as a compound proposition whose truth value is determined by the truth value of its component proposition / propositions.**

(2) Non – truth functional compound proposition :

There are some compound propositions whose truth value is not determined by the truth value of its component proposition / propositions.

Such compound propositions are called **Non – truth functional compound proposition**

Example : I believe that Soul exist.

Here the **component proposition** "Soul exist" may be either true or false.

Whatever may be the truth value of the **component proposition**, the truth value of the compound proposition does not get affected.

If the proposition, 'I believe that Soul exit" is say true, then whether the component proposition "Soul exist" is true or false. The truth value of the compound proposition will remain true.

Hence, It is a Non Truth functional compound proposition.

Thus **Non – truth functional compound proposition is defined as a compound proposition whose truth value is not determined by the truth value of its component proposition / propositions.**

Types of truth functional compound proposition :

On the basis of the connectives which combine the components in truth functional propositions, we get five types of truth functional compound proposition.

(1) **Negative** proposition

Example : This book is **not** interesting.

(2) **Conjunctive** proposition

Example : This book is interesting **and** informative.

(3) **Disjunctive** proposition

Example : **Either** this book is interesting **or** informative.

(4) **Material Implicative** or conditional proposition –

Example : **if** this book is interesting **then** people will buy the book.

(5) **Material Equivalent** or Bi – conditional proposition –

Example : People will buy this book **if and only if** it is interesting.

2.3 Symbolization of proposition :

Need, uses and importance of symbolization.

Symbolization is necessary because arguments are expressed in language. The use of symbols is not misleading but it helps us to reason correctly.

There are certain defects of natural language as follows.

(1) use of ambiguous words and vague words.

(2) use of misleading idioms.

(3) confusing metaphorical style.

The symbolic language is free from the above mentioned defects.

Logic is concerned with arguments. Arguments contain propositions or statements as their premises and conclusion. Arguments may be valid or invalid. To determine the validity of the arguments we have to use certain logical procedures. These procedures can not be applied directly to the propositions with ordinary language. Logicians have developed specialized techniques to bring out the form of the proposition. It is done by symbolizing propositions.

Deductive Logic is concerned with the form of an argument and not with the content of argument. It is form of a proposition. This can be done by symbolization.

Use of symbols is convenient and advantageous, for better understanding of arguments and drawing of inference from it.

Significance of symbolization in Logic –

(1) It helps to focus on what is important in an argument and to ignore unnecessary details, thus helps to decide it's validity easily.

(2) It helps to understand the logical structure of propositions and arguments more clearly.

(3) It prevents confusion of vague and ambiguous words.

Symbols are kind of short – forms. In a natural language a proposition or an inference has a much longer expression. When we use symbols the expression becomes much more shorter.

For symbolizing of truth functional compound propositions. We need certain symbols. They are –

(1) Propositional Constant

(2) Propositional Variable

(3) Propositional Connective or Operator

(4) Brackets

(1) Propositional Constant :

Propositional constant is defined as a symbol, which stands for a specific (or particular) proposition as a whole. They are called constants because they have definite meaning. The capital letters from **A to Z** (English alphabet) are used as propositional constants. We are free to use any propositional constants for symbolizing of a proposition.

Example : Yogasanas act as bridges to unite the body with the mind.

The above proposition can be symbolized as **"A"** or by any other capital letter which will stand for the whole proposition.

When an argument contains more number of propositions as components we have to observe following **conditions or restrictions.**

(1) The same propositional constant is to be used for symbolizing a proposition if it occurs again in the same argument (or in the same compound propositions)

(2) The same propositional constant can not be used for different propositions in the same argument. (or in the same compound proposition)

Example : Santosh will take salad or sandwich.

Santosh will not take salad.

Therefore, Santosh will take sandwich.

In the **above example** for the proposition "Santosh will take salad." we will choose the propositional constant "S" and for the proposition "Santosh will take sandwich" we cannot use the same propositional constant "S". (as per restriction no.2) so we will have to use different propositional constant, like "D"

Example :

The **first proposition (premise)** is

Santosh will take salad or sandwich

The **symbolization** of this proposition will be

S or D

The **second proposition (premise)** is

Santosh will not take salad.

The **symbolization** of this proposition will be

Not **S**

The **third proposition (conclusion)** is

Santosh will take sandwich

The **symbolization** of this proposition will be

Therefore **D**

Thus the argument may now will be symbolized thus:

S or D
Not S
Therefore D

(2) Propositional Variable :

Propositional variable is defined as a symbol which stand for any proposition whatsoever. Small latter p, q, r, s …….. (English alphabet) are used as propositional variable. Propositional variable does not stand for any specific proposition. It only marks or indicates the place of proposition.

For Example : The expression "if p then q" indicates that "p" stands for any proposition and "q" stands for any other proposition and these two different propositions are connected by the expression "if…………then".

A propositional variable is a symbol used to substitute a proposition.

When an argument form contains more number of propositions as components we have to observe following **conditions or restrictions.**

(1) The same propositional variable is to be substituted by the same proposition if it occurs again in the same argument (or in the same compound proposition)

(2) The same propositional variable can not be substituted by different propositions in the same argument. (or in the same compound proposition)

In an argument of the following form, for instance by substituting any proposition for "p" and any other proposition for "q" we will get innumerable arguments.

Example : if p then q

Not q

Therefore Not p

Example No : 1

If a figure is a square then it has four sides.

The figure does not have four sides.

Therefore the figure is not a square.

Example No : 2

If you have a password then you can log on to the network.

You can not log on to the network.

Therefore, you do not have a password.

We can substitute any proposition for a propositional variable, it is therefore said to be a **place marker / place holder** or **dummy letter.**

Activity 9. Read the following arguments forms carefully and construct arguments form it.

(1) Either p or q	(2) If p then q	(3) If p then q
Not p	p	If q then r
Therefore q	Therefore q	Therefore If p then r

Propositional Connective	symbol	Name of the symbol
(1) Not	~	Tilde / Curl
(2) And	•	Dot
(3) Either ………. Or	∨	Wedge
(4) If ……….. then	⊃	Horse - shoe
(5) If and only if then	≡	Tripple Bar

(3) Propositional connective – (Truth – Functional logical operator) –

Propositional connective is defined as an expression which operates on proposition or propositions or they connects two propositions in a truth functional compound proposition. There are five expressions which connect component or components in a truth functional compound proposition. The name of the symbols for five connectives, are given below. These symbols are also called **logical constants / operators.**

The propositional connective "not" operates on one proposition only.

Therefore it is known as **Monadic operator**.

On the other hand, the last four connectives or operators namely [and, either…….or, if….. then……, if and only if…… then…..] connects two propositions. Therefore, they are known as **Binary or Dyadic operators.**

Importance of Bracket (s) in symbolization –

In language punctuation is requires to make complicated statements clear.

Punctuation are a mark such as full stop, comma or question mark, exclamation mark, semicolon, inverted comma etc. which are used in writing to separate sentences and their elements and to clarify their meaning.

Example : (from English Language)

Why we need commas because
"I like cooked vegetables, fruits and dogs."
is not same as
"I like cooked vegetables fruits and dogs."

In mathematics, to avoid ambiguity and to make meaning clear, punctuation marks appear in the form of brackets.

Example : $6 + 7 \times 8$

It could be $6 + (7 \times 8)$ or $(6 + 7) \times 8$

So, in Logic some punctuation marks are equally essential, to clear the complicated propositions. In symbolic Logic parentheses, brackets and braces are used as a punctuation marks.

(1) Parentheses : It is a symbol '( )' that is put around a word or a phrase or a sentence.

Example : $(p \bullet q) \supset r$

(2) Box Brackets : It is used to enclose words or figures. In logic it is used to group expressions that include parentheses.

Example : $[\,(p \bullet q) \vee (q \bullet p)\,] \equiv r$

(3) **Braces :** It is used to group expressions that include box brackets. Example: { }

Example : ~{[(p • q) ∨ (q • p)] ≡ p }

Truth functional compound propositions

On the basis of five propositional connectives, there are five types of Truth functional compound propositions. They are as follows –

(1) Negative proposition

(2) Conjunctive proposition

(3) Disjunctive proposition

(4) Material Implicative or Conditional proposition

(5) Material Equivalent or Biconditional proposition

(1) Negative proposition :

When any proposition is negated or denied we get negative proposition. Negation is commonly expressed in English language by the word **"Not"**. But a proposition can be negated with the help of words like **it is not the case that, it is not true that, it is false that, none, never.**

Example :

(1) Sadanand is **not** a mathematician.

(2) **It is false** that Ajit is taller than Rajesh.

(3) **It is not true** that Urmila is a magician.

(4) **It is not the case that** Ajay is a singer.

In Logic we use symbols for propositional connectives as well as propositions. For the connective "Negation" or the word "not" the symbol " ~ " is used.

This symbol is called as **"Tilde" or "Curl"**

Using the symbol "~" for negation and the propositional variable "p" for any proposition whatsoever, we get the form of negative propositions as follows :

~ p

Symbolization :

Example : Sadanand is not a mathematician.

Step 1: The above example consists of **one proposition** and one propositional operator.

Underline the proposition and put a propositional operator in the **box**.

So we will get following expression:

Example :

<u>Sadanand is</u> $\boxed{\textbf{not}}$ a <u>mathematician</u>.

~ S

Thus the form of negative proposition is '~ p'. This is read as 'Not p'.

Always Remember :

~ Sign to be written before the letter or on the left hand side of the letter.

~ P $\boxed{✓}$ P ~ $\boxed{×}$

Truth value for negation –

Negation is also known as contradictory function.

A negative statement is true when its component proposition is false and vice versa.

Basic truth table for negation :

~	P
F	T
T	F

(2) Conjunctive Proposition – (conjunction)

When two propositions are joined together by truth – functional connective **"and"** it is called a conjunctive proposition.

The components of conjunctive proposition are called as **Conjuncts**.

The word "and" is called dyadic connective or binary operator, as it connects two propositions.

Example : Be good and you will be happy.

The above example consists of two propositions –

(1) Be good

(2) You will be happy.

These are connected by the word "and".

Often we use word such as **but, though, although, while, yet, also, still, nevertheless, however, moreover, further, as well as, neither....... nor, in the conjunctive sense.**

Example :

(1) The lion is called king of the forest **and** it has a majestic appearance.

(2) I want to go to the party, **but** I am tired.

(3) Gauri is playing, **while** Varsha is studying.

(4) The couch was shouting, **yet** the players remained noisy.

(5) Hemangi kept working even **though** she was tired.

(6) It's a small house **still** it is spacious.

(7) Chocolates are **neither** nutritious **nor** good for teeth.

(8) Mr. Patil is a politician **and** Sai baba is a saint.

Symbolic form of conjunctive proposition will be as follows :

Example : Be good and you will be happy.

To symbolize a propositional operator "And" we can use symbol (•)

Symbolic from of conjunctive proposition is as follows:

'p • q'

Example : Sugandha is a mother and a grandmother.

Above proposition consists of two parts (components)

(1) Sugandha is a mother.

(2) Sugandha is a grandmother.

These two parts or components of a conjunctive proposition are called as **Conjuncts** in the language of Logic.

Sugandha is a mother	and	a grandmother.
(First conjunct)		**(Second conjunct)**

Thus symbolization of above proposition is M • G

Thus the form of conjunctive proposition is 'p • q'. It is read as ' p and q'.

Truth Value :

A conjunctive proposition is a kind of truth functional compound proposition. Hence, the truth value of a conjunctive proposition depends on its components i.e. conjuncts.

A conjunctive proposition is true only when both the conjuncts are true otherwise it is false.

Basic truth table for conjunction :

p	•	q
T	**T**	**T**
T	F	F
F	F	T
F	F	F

(3) Disjunctive proposition – (Disjunction)

When two propositions are joined together by truth functional connective …. **'either ……. …… or'**, it is called a disjunctive proposition. The word "either …… Or" is called dyadic connective or binary operator, which connects two statements. The components of disjunctive proposition are called as **"Disjuncts"**.

Example :

(1) **Either** I will go to Prague **or** Vienna.

(2) **Either** she is weak **or** coward.

(3) The car is **either** blue **or** red.

Example :

Either he is rich or poor.

In the above example there are two propositions.

(1) he is rich

(2) he is poor

These two propositions are connected by truth – functional connective or logical connective" Either…………..Or".

Form of the disjunctive proposition is "p ∨ q". This is read as "p or q".

Symbolization :

Either he is rich	or	he is poor.
Proposition	Logical connective or operator	Proposition
(First disjunct)		(Second disjunct)
R	∨	P

Therefore, symbolization of the above proposition will be:

R ∨ P

Form of disjunctive proposition is 'p ∨ q'. This is read as 'p or q'.

Truth Value :

A disjunctive proposition is a kind of truth functional compound proposition. Hence, the truth value of a disjunctive proposition depends on its components i.e. disjuncts.

A disjunctive proposition is false, only when both the disjuncts are false otherwise it is true.

Basic truth table for disjunction :

p	∨	q
T	T	T
T	T	F
F	T	T
F	**F**	**F**

Disjunctive proposition may be used in the inclusive (weak) sense or exclusive (strong) sense.

(1) The Inclusive or weak sense of "OR" –

When both the disjuncts can be true, the word or is said to be used in inclusive sense.

Rajvi is either a mother or an actress.

In the above proposition there are two disjuncts.

(1) Rajvi is a mother.

(2) Rajvi is an actress.

Both these disjuncts can be true together because a person can be both a mother and an actress.

In other words the statement can be interpreted as "either p or q or both". Means "p" alone can be true, "q" alone can be true and both can be true together but cannot be false.

(2) Exclusive or strong sense of OR :

When both the disjuncts cannot be true together, the word "Or" is said to be used in exclusive sense.

Example : Either it is a sparrow or a crow.

In the above proposition there are two disjuncts.

(1) A bird is a sparrow.

(2) A bird is a crow.

Both these disjuncts cannot be true together. If one is true, other is necessarily (exclusively) false.

In other words this can be interpreted as, either "p" is true or "q" is true but both cannot be true together. i.e. if a bird is a sparrow then it cannot be a crow or vice versa.

In logic, disjunctive proposition is used in the inclusive sense only.

(4) Material Implicative or Conditional proposition –

When two propositions are joined together by truth functional connective **if …… then …….** it is called an implicative proposition.

Example :

(1) **If** you want a good pet **then** you should get a dog.

(2) **If** my car is out of fuel **then** it will not run.

(3) **If** a figure is a rhombus **then** it is not a rectangle.

4] **If** you do all the exercises in the book, you will get full marks in the exam.

5] **If** it is a molecule **then** it is made up of atoms.

(sometimes ' , ' (coma) is used instead of word "then")

Words indicating implicative proposition – The expression like "if ………then", **"in case"**, **"had it"**, **"unless" (if not)** indicate that the proposition is a conditional proposition.

If it rains	then	the trains will run late.
Proposition 1		Proposition 2

If ……….. then ………. is logical operator.

The symbolic expression :

$$R \supset T$$

Thus, the form of the implicative proposition is " p ⊃ q. This is read as "if p then q" or "p implies q".

An **implicative proposition** is also called as **conditional proposition** because they state the condition and its consequences.

The proposition that **states the condition** is called as **antecedent** and the proposition that **states result** is called as **consequent**.

Example :

If she is tall	then	she can become a model.
Condition		Result
antecedent		consequent

Truth value :

An implicative proposition is false only when its antecedent is true and its consequent is false. Otherwise it is always true.

Basic truth table for material implication :

P	⊃	q
T	T	T
T	**F**	**F**
F	T	T
F	T	F

(5) Material Equivalent or Biconditional proposition. –

A biconditional proposition is a compound proposition in which two component propositions materially imply each other.

When two propositions are joined together by truth – functional connective **if and only if …. then…**, it is called as an material equivalent proposition.

Example :

(1) You can take a flight **if and only if** you buy a ticket.

(2) Two angles are congruent **if and only if** their measurements are equal.

(3) You can enter the theatre **if and only if** you have the entry pass.

(4) **If and only if** you study hard, you will pass.

Always remember

‘ ,’ (coma) is used to make the statement meaningful.

In a conditional statement antecedent implies consequent but consequent does not imply the antecedent. In biconditional statement, however, the first component implies the second and the second component also implies the first.

The expression “if and only if ” indicates that the statements is a biconditional statement.

Example :

Birds fly	if and only if	sky is clear.
Proposition 1	logical connective	proposition 2

The symbolic expression :

$$B \equiv S$$

OR

$$S \equiv B$$

Thus, the form of the biconditional statement is **“ p ≡ q”**. This is read as “if and only if p then q “or” “p is materially equivalent to q”.

Truth value :

A biconditional proposition is true if and only if both the components have the same truth value. i.e. either both the components are true or both the components are false. Otherwise the statement is false.

Basic true table for material equivalance :

p	≡	q
T	**T**	**T**
T	F	F
F	F	T
F	**T**	**F**

Activity : 10

(1) I will go to a mall.

(2) I will go to a movie.

(3) I will go to gym

Use the above propositions and construct 5 types of truth functional propositions.

2.4 Symbolising compound proposition :

(1) Roses are red and Jasmines are white.

Proposition 1	logical connective	proposition 2
R	•	J

So, we will answer this with the help of two steps.

Symbolization : R • J

Kind of proposition : Conjunctive proposition.

(2) He is poor but not hardworking.

Proposition 1	logical connective	proposition 2
P	• ~	H

Symbolization : P • ~ H

Kind of proposition : Conjunctive proposition

(3) Mira is not both a good singer and a good actress.

Symbolization : ~ (S • A)

Kind of proposition : Negative proposition.

(4) **If** the road is wet, **then either** it has rained today **or** the fire truck spilled water on the road.

Symbolization : W ⊃ (R ∨ F)

Kind of proposition : Implicative or conditional proposition.

(5) He goes to play a match if and only if it does not rain.

Symbolization : ~ R ≡ M

Kinds of proposition : Equivqlent or Bi – conditional proposition.

(6) It is false that if and only if I will go to Australia, I will earn money.

Symbolization : ~ (A ≡ M)

Kind of proposition : Negative proposition.

(7) Either Sun is a star or not a star.

Symbolization : S V ~ S

Kind of proposition : Disjunctive proposition

(8) Neither it is hot nor cold today.

Symbolization : ~ H • ~ C

Kind of proposition : Conjuctive proposition

(9) If fast food is not healthy then one must not eat it.

Symbolization : ~ H ⊃ ~ E

Kind of proposition : Implicative or Conditional proposition.

(10) A living being is either mortal or immortal.

Symbolization : M ∨ I

Kind of proposition : Disjunctive proposition.

Monadic operator	Dyadic operator
(1) It operates on one proposition.	(1) It operates on or connects two propositions.
(2) ~ is a monadic operator.	(2) •, ⊃, ∨, ≡ are dyadic or binary operator.

Always remember

All dyadic connectives are always placed in between the two component proposition.

p • q	✓	• p q	×
p ∨ q	✓	∨ p q	×
p ⊃ q	✓	⊃ p q	×
p ≡ q	✓	≡ p q	×

Summary

Proposition : A proposition is a sentences which is either true or false. Most logicians use the words, "proposition" and " statements" in the same sense. If a proposition represents facts, it is true, Otherwise, it is false.

Proposition and a sentence : A proposition is expressed in the form of a sentence. However, proposition differs from a sentence.

In /modern propositional logic, proposition are classified into –

(1) Simple Proposition :

(a) Subject less proposition

(b) Subject – Predicate Proposition

(c) Relational Proposition –

(d) Class – membership proposition

(2) Compound Proposition –

(a) Truth – functional compound proposition.

(b) Non – truth functional compound proposition.

Classification of Truth functional compound proposition –

(1) Negative proposition

(2) Conjunctive proposition

(3) Disjunctive proposition

(4) Material Implication or conditional proposition

(5) Material Equivalent or Biconditional proposition

Modern Logicians use constants, variables, logical operators and brackets for symbolizing propositions.

Exercises

Q. 1. Fill in the blanks with suitable words from those given in the brackets:

1. ……… is a basic unit of Logic. *(Sentence / Proposition)*
2. A proposition is conveyed through a ……. *(Statement / Sentence)*
3. If a proposition represents a fact, it is said to be ……… *(False / True)*
4. Only ……… sentences are proposition. *(Declarative / Exclamatory)*
5. ……… proposition does not contain any other proposition as its component. *(Simple / Compound)*
6. A, B, C, D are …….. *(Propositional Constant / Propositional Variable)*
7. ' • ' is a ……… connective. *(Binary / Monadic)*
8. In logic disjunctive proposition is used in the ………… sense only. *(Exclusive / Inclusive)*
9. An implicative proposition is false when its ….. is true and ….. is false. *(Consequent / Antecedent)*
10. The symbol used for a biconditional proposition is ….. . *(≡ / ∨)*

Q. 2. State whether the following statements are true or false.

1. The premise and conclusion are known as prppositions.
2. Every sentence asserts a proposition.
3. A proposition is false, if it stands for actual state of affairs.
4. When we negate simple proposition, we get a compound proposition.
5. A conjunctive proposition is false, when any one conjunct is false.
6. A variable is not proposition but is a "place holder" for any proposition.
7. The symbol ⊃ is a logical operator.
8. A proposition is neither true nor false.
9. In class – membership proposition, predicated is general.
10. The components of disjunctive proposition are called as disjuncts.

Q. 3. Match the columns :

	Group (A)		Group (B)
1.	Sentence	a.	v
2.	Dyadic Connective	b.	Negation
3.	Strong Disjunction	c.	Conjunctive proposition
4.	~ (p ∨ q)	d.	Either he is tall or short.
5.	And, yet, still etc.	e.	physical existence

Q. 4. Give logical terms for the following :

1. It is a meaningful group of words in a grammatical order.
2. A proposition asserts that an individual is a member of a class.
3. It is a symbol which stands for any proposition whatsoever.
4. The components of disjunctive proposition.
5. Truth or falsity of proposition.

Q. 5. Give reasons for the following :

1. '~' is called a monadic operator.
2. When we negate simple proposition we get compound proposition.
3. Equivalent proposition is also called as Bi-conditional proposition.
4. "Suresh is either a doctor or a teacher' is an inclusive sense of disjunction.
5. When we use symbols, the expression becomes much more shorter.

Q. 6. Explain the following :

1. Basic unit of logic
2. Conjunctive proposition
3. Logical operator.
4. Truth – functional compound proposition

Q. 7. Answer the following questions.

1. Explain the difference between proposition and sentence.
2. All propositions are sentences but all sentences are not proposition. Explain.
3. What are the restrictions on a propositional constants? Explain with example.
4. When a conjunctive proposition is true and false?
5. What is the difference between conditional proposition and Bi-conditional proposition.

Q. 8. Symbolize the following propositions using appropriate symbols given in the bracket and identify their kind:

1. He is creative and hardworking. (C, H)
2. If a student completes academic course then he will be graduated. (A, G)
3. It is false that parking is prohibited in this area. (P)
4. If and only if Viraj scores double century, we will win the match. (V, M)
5. This tour is not both safe and exciting. (S, E)
6. It is not the case that, the professor will take a leave if and only if the administration allow him. (P, A)
7. Pizza and Burger is a perfect combination (P, B)
8. She is neither well behaved nor humble. (W, H)
9. I will buy this dress if and only if it is not expensive. (D, E)
10. Puranpoli is delicious, but it is not good for diabetic patient. (P, D)
11. Either Danashree is a talented musician or she is not. (M)
12. If Ramesh was warm and caring person then I am an alien from outer space. (W, C, A)
13. B.E.S.T. is the heart of Mumbai city. (M)
14. If “Ted Talks’ are informative and inspirational then people will follow it. (I, N, P)
15. She is simple yet presentable. (S, P)
16. If the road is wet then either it rains today or the water tanker spill water on the road. (R, T, W)
17. You are not allowed to take leave without permission. (L)
18. It is not the case that Bhalchandra is a superstar and not a superstar. (S)
19. Either cat fur or dog fur was found at the scene of the crime. (C, D)
20. Siddharth Mukherjee is a cancer physician and winner of the 2011 Pulitzer Prize. (P, W)
21. It is false that if Ranjit is a good singer then he will be a great musician. (G, M)
22. If company does not increase the salary of the workers then the union will go on strike. (S. U)
23. The Young inventor Richard Turere invented “lion lights” an elegant way to protect his family’s cattle from lion attacks. (E)
24. Himalaya is snowy and majestic. (S. M)
25. If Mom’s brinjal plants are ruined then an elephant was walking in her garden. (B, E)
26. Sujata will eat the fruit if and only if it is mango. (F, M)
27. If sharks are disturbed then they become aggressive. (D, A)
28. It is true that poverty is a worst enemy of man. (P)

29. Either no students are interested in giving feedback or it is not the case that administration requires the students' feedback. (F, A)

30. If hydrochloric acid (HCI) and sodium hydroxide (NaOH) are combined, then table salt (NaCI) will be produced. (H, S, T)

31. Success does not mean a lot of money or gaining lot of fame. (M,G)

32. If the pavement is **not** wet, then it did **not** rain. (W, R)

33. Cats are good pets and they are affectionate. (P, A)

34. Omkar ran fast but he missed the train. (F, T)

35. If Seema is in Denmark, then she is in Europe and If Seema is not in Europe, then she is not in Denmark. (S. E)

36. **Memory Banda** is Malawian children's rights activist who has drawn international attention for her work in opposition to child marriage. (M)

37. If a triangle is equilateral then its angles all measure 60 and if all the angles of a triangle measure 60 then the triangle is equilateral. (T, A)

38. If I pass then I will have a party and if I fail then also I will have a party. (P, T, F)

39. Swayam talks is not just another talk series but its presentation make it a unique concept. (T, U)

40. Either Leena will learn music or dance. (M, D)

Activity 11 : Complete the following table

Sr. No.	Kinds of Proposition	Propositional connective	symbol
1.			' $\supset$ '
2.	Conjunctive Proposition		
3.		Either ….. Or …..	
4.	Negative Proposition		
5.			$\equiv$

❖ ❖ ❖

Chapter 3 Decision Procedure

The concept of decision procedure is predominantly concerned with the concept of decidability.

DO YOU KNOW THAT ....

Logic can help us to determine correctness of certain reasoning by formulating tables.

Some statement forms are always false.

When you ask your friend, would you go to London or Paris, he can choose both.

3.1 CONCEPT OF DECISION PROCEDURE

In the earlier chapter, we have studied about the nature of proposition, its kinds and its basic truth values. In this chapter, we are going to study the procedure for deciding the validity of arguments. In logic, we use the decision procedure (method) to decide whether a truth functional form is Tautologous, Contradictory or Contingent. It also tests whether an argument is valid or invalid. Decision Procedure may be defined as a method of deciding whether an object belongs to a certain class.

There are five types of Decision Procedures:

(1) Truth Table

(2) Shorter Truth Table

(3) Truth Tree

(4) Conjunctive Normal Form

(5) Disjunctive Normal Form

In this text we shall study the method of constructing Truth Table as a decision procedure.

Characteristics of decision procedure :

A decision procedure must be effective, to be an effective decision procedure certain conditions need to be fulfilled. –

(1) Reliable : A decision procedure must be reliable. A reliable procedure is one which always gives a correct answer, provided we use the method and rules correctly.

(2) Mechanical : A decision procedure is mechanical ie just by following certain steps in a certain order one can get an answer. There is no scope for one's imagination and intelligence.

(3) Finite : A decision procedure must be finite ie it should have limited number of steps. There should be a last step for getting the answer.

3.2 NATURE OF TRUTH TABLE

Truth table is one of the decision procedures. **A truth table is defined as a tabular way of expressing the truth value of expressions containing propositional connectives (a truth functionally compound statement).**

Procedure of Construction of a truth table (for truth – functional statement form)

(1) To construct a truth table we shall first make two columns: one on the left hand side for the matrix and the other on the right hand side for the truth – functional form for which the truth table is constructed.

Example : $(q \vee p) \equiv [(p \bullet q) \supset p]$

Matrix	Truth – Functional Form
	$(q \vee p) \equiv [(p \bullet q) \supset p]$

The first step is to write down the Truth functional form in the column for Truth functional form.

(2) The second step is to write down in the matrix column all the distinct variables occurred in the truth functional form.

In above example, there are two, distinct variables 'p' and 'q'. So we write them as follows.

Matrix	Truth – Functional Form
p q	(q ∨ p) ≡ [(p • q) ⊃ p]

(3) The third step is to determine the number of rows the truth table will have. The number of rows depends upon the number of propositional variables, occurred in the truth functional form. The simple formula is,

2^n = Number of rows.

(n = Number of distinct variables occurring in the expression)

No. of distinct variable		No. of rows
2^1	2 x 1	2
2^2	2 x 2	4
2^3	2 x 2 x 2	8
2^4	2 x 2 x 2 x 2	16
2^5	2 x 2 x 2 x 2 x 2	32

Activity : 1

2^6 = ______ = ______

2^7 = ______ = ______

(4) Fourth step is to construct the matrix.

A matrix consists of all the possible combinations of the truth values of the propositional variables in the truth function or argument.

(a) Matrix for one variable

Example : (p • p) ∨ p

Matrix	Truth – Functional Form
p	(p • p) ∨ p
T	
F	

(b) Matrix for two variables:-

Example : (p ∨ q) ⊃ (q ⊃ p)

Matrix	Truth – Functional Form
p q	(p ∨ q) ⊃ (q ⊃ p)
T T	
T F	
F T	
F F	

(c) Matrix for three variables:-

Example : p ≡ (q • r)

Matrix	Truth – Functional Form
p q r	p ≡ (q • r)
T T T	
T T F	
T F T	
T F F	
F T T	
F T F	
F F T	
F F F	

Always Remember

The propositional variables are written in the alphabetical order in the matrix.

E.g. (r ∨ q) • r

Matrix		Truth – Functional Form
q	r	(r ∨ q) • r
T	T	
T	F	
F	T	
F	F	

Activity 2 : 1

Construct matrix for 4 variables i.e. p, q, r, s

Construct matrix for 5 variables i.e, p, q, r, s, t.

Activity 2:2 Complete the matrix column

Activity : 1

$\boxed{r}$	(r ⊃ r) ∨ (r • r)

Activity : 2

q $\boxed{}$	(t • q) ≡ (q ∨ t)

Activity : 3

$\boxed{}$ $\boxed{}$	(p ∨ s) ≡ (p ⊃ s)

Activity : 4

$\boxed{}$ $\boxed{}$ $\boxed{}$ $\boxed{}$	(r ⊃ s) • (p ≡ t)

Let us continue with same example

Matrix		Truth – Functional Form
p	q	(q ∨ p) ≡ [(p • q) ⊃ p]
T	T	
T	F	
F	T	
F	F	

(5) Let us construct the truth table. In the above truth – functional form, there are two distinct variables i. e,. 'p' and 'q', wherever 'p' occurs in the truth functional form we shall write down the truth values written under 'p' in the matrix. Then wherever 'q' occurs in the truth function we shall write down the truth – values written under 'q' in the matrix. After assigning the values to 'p' and 'q' variables, the truth table will be as follows.

Matrix		Truth – Functional Form				
p	q	(q ∨ p) ≡ [(p • q) ⊃ p]				
T	T	T	T	T	T	T
T	F	F	T	T	F	T
F	T	T	F	F	T	F
F	F	F	F	F	F	F

(6) In the previous chapter, we have learnt the basic truth values of compound propositions. Accordingly we shall now determine the truth value of the truth functional form.

Example : (q ∨ p) ≡ [(p • q) ⊃ p]

❖ In our expression, '≡' is the main connective.

(q ∨ p) ≡ [(p • q) ⊃ p]

- First we shall find out the truth value of the expression on the left – hand side of the truth functional form, i.e. disjunction between 'q' and 'p' as follows.

Matrix		Truth – Functional Form
P	q	(q ∨ p) ≡ [(p • q) ⊃ p]
T	T	T **T** T T T T
T	F	F **T** T T F T
F	T	T **T** F F T F
F	F	F **F** F F F F

- Then, we shall determine the truth value of the expression on the right – hand side of the truth functional form i.e the value of conjunction between 'p' and 'q' as follows.

Matrix		Truth – Functional Form
P	q	(q ∨ p) ≡ [(p • q) ⊃ p]
T	T	T **T** T T T T T
T	F	F **T** T T F F T
F	T	T **T** F F F T F
F	F	F **F** F F F F F

- Let us determine the truth value of the conditional statement between conjunction i.e., p • q and variable 'p' to the right side.

Matrix		Truth – Functional Form
P	q	(q ∨ p) ≡ [(p • q) ⊃ p]
T	T	T **T** T T T T **T** T
T	F	F **T** T T F F **T** T
F	T	T **T** F F F T **T** F
F	F	F **F** F F F F **T** F

- Finally, let us determine the truth value of material equivalent statement which is the main connective i.e. between (q ∨ p) and [(p • q) ⊃ p] which will give us the truth value of truth – functional form under all possibilities. We need to consider disjunction in the left bracket and implication in the right bracket. Taking these two values, we will determine the value of equivalence which is the main connective.

  Thus, the final truth table will be as follows:

Matrix		Truth – Functional Form
p	q	(q ∨ p) ≡ [(p • q) ⊃ p]
T	T	T **T** T **T** T T T **T** T
T	F	F **T** T **T** T F F **T** T
F	T	T **T** F **T** F F T **T** F
F	F	F **F** F **F** F F F **T** F

  This truth table shows that under the main connective, only in one possibility i.e, in the fourth row, the truth functional form is false. In remaining possibilities it is true.

- Let us understand truth table with more examples.

Example 2 : (~ r • ~ p) ⊃ (r ∨ ~ p)

Matrix		Truth – Functional Form
p	r	(~ r • ~ p) ⊃ (r ∨ ~ p)
T	T	F T **F** F T **T** T **T** F T
T	F	T F **F** F T **T** F **F** F T
F	T	F T **F** T F **T** T **T** T F
F	F	T F **T** T F **T** F **T** T F

Example 3 : ~ (t ∨ q) • ~ (~ t • ~ q)

Matrix	Truth – Functional Form
q t	~ (t ∨ q) • ~ (~ t • ~ q)
T T	**F** T T T **F** **T** F T F F T
T F	**F** F T T **F** **T** T F F F T
F T	**F** T T F **F** **T** F T F T F
F F	**T** F F F **F** **F** T F T T F

Activity : 3

Complete the following tables

Matrix	Truth – Functional Form
q	(q ⊃ ~ q) • ~ q
T	☐ **F** ☐ ☐ F ☐ T
F	F ☐ ☐ ☐ ☐ ☐ ☐

Matrix	Truth – Functional Form
p s t	t ≡ (p ∨ s)
T T T	☐ ☐ T **T** ☐
T T F	☐ **F** ☐ ☐ ☐
T F T	**T** ☐ ☐ ☐ ☐
T F F	☐ ☐ T ☐ ☐
F T T	☐ ☐ F ☐ ☐
F T F	**F** ☐ ☐ **T** T
F F T	☐ ☐ ☐ ☐ F
F F F	☐ **T** ☐ ☐ ☐

Concept of Tautology, Contradiction and Contingency

The truth functional statement forms are broadly classified into three kinds. They are Tautology, Contradiction and Contingency.

(1) Tautology :

A tautology is a truth functional statement form which is "True" under all truth possibilities of its components. It means that in the truth table for a tautology truth value "True" appears under the main connective in all the rows. Thus, tautology is a statement form which has all true substitution instances.

Example : (p • ~ p) ⊃ ~ p

Matrix	Truth - Functional Form
p	(p • ~ p) ⊃ ~ p
T	T **F** F T **T** **F** T
F	F **F** T F **T** **T** F

In the above statement form, truth value T's appear under the main connective, so the given truth functional form is tautology.

(2) Contradiction :

A contradiction is defined as a truth functional statement form which is 'False' under all truth possibilities of its components. It means that in the truth table for a contradiction truth value 'False' appears under the main connective in all the rows. A contradiction is a statement form which has only 'False' substitution instances.

Example : (p ≡ ~ p) • (~ p ⊃ p)

Matrix	Truth - Functional Form
P	(p ≡ ~ p) • (~ p ⊃ p)
T	T **F** F T **F** F T **T** T
F	F **F** T F **F** T F **F** F

In the above statement form, truth value F' s appears under the main connective, so the given truth functional form is contradiction.

(3) Contingency :

A contingency is defined as a truth functional statement form which is 'True' as well as 'False' under some truth possibilities of its components. It means that in the truth table for contingency truth value 'True' as well as 'False' appears under the main connectives in truth possibilities. Thus contingency is a statement form which has some true as well as false substitution instances.

Example : (p • ~ p) ≡ (p ⊃ ~ p)

Matrix	Truth - Functional Form
P	(p • ~ p) ≡ (p ⊃ ~ p)
T	T **F** F T **T** T **F** F T
F	F **F** T F **F** F **T** T F

In the above truth functional statement form T's and F' s appears under the main connective. So, the given propositional form is a contingency.

Relation between Tautology, Contradiction and Contingency:

(1) Denial of tautology leads to contradiction.

For Example : the truth – functional form '(p • p) ⊃ p ' is a tautology. Its denial i.e. ~ [(p • p) ⊃ p]is a contradiction.

(2) Denial of contradiction leads to tautology.

For Example : the truth – functional form ' (p • ~ p)' is a contradiction. Its denial i.e. ~ (p • ~ p) is a tautology.

(3) Denial of contingency leads to contingency.

For Example : the truth -functional form (~p •q) is a contingency. Its denial i.e. ~ (~p •q) is a contigency.

Let us now construct truth table for some truth – functional statement forms and determine whether they are tautology, contradiction or contingency.

Example 1 : : ~ [p • (p ∨ ~ p)] ⊃ (p ⊃ p)

Matrix	Truth Functional Form
p	~ [p • (p ∨ ~ p)] ⊃ (p ⊃ p)
T	**F** T T T T F T **T** T **T** T
F	**T** F F F T T F **T** F **T** F

Example 2 :

Matrix	Truth - Functional Form
p q r	(p ⊃ q) ∨ r
T T T	T **T** T **T** **T**
T T F	T **T** T **T** **F**
T F T	T **F** F **T** **T**
T F F	T **F** F **F** **F**
F T T	F **T** T **T** **T**
F T F	F **T** T **T** **F**
F F T	F **T** F **T** **T**
F F F	F **T** F **T** **F**

Example 3 :

~ (q ∨ p) • ~ (~ q • ~ p)

Matrix	Truth 0 Functional Form
p q	~ (q ∨ p) • ~ (~ q • ~ p)
T T	**F** T T T **F** **T** F T F F T
T F	**F** F T T **F** **T** T F F F T
F T	**F** T T F **F** **T** F T F T F
F F	**T** F F F **F** **F** T F T T F

Activity : 4

State whether the above statement forms are tautology, contradiction or contingency with reason.

Activity : 5

Construct the truth table for the following truth – functional forms and determine whether they are tautologies, contradictions or contingencies.

1. $(\sim q \supset \sim p) \equiv (p \supset q)$
2. $p \vee (q \bullet r)$
3. $(\sim p \bullet p) \vee p$

3.3 Truth table as a decision procedure for arguments

An a argument is a group of statements. An argument consists of simple and truth – functionally compound propositions.

Let's now examine a truth table method to determine an argument form to be valid or invalid.

Example :

Amita is intelligent and courageous.

Amita is intelligent.

Amita is courageous.

Therefore either Amita is intelligent or courageous.

(I, C)

Let's symbolize the given argument.

❖ Symbolization of an argument

(1) $I \bullet C$

(2) I

(3) C

$\therefore I \vee C$

❖ Now we will convert the given symbolized argument in argument form.

(1) $p \bullet q$

(2) p

(3) q

$\therefore p \vee q$

❖ Now let us construct a truth table for a given argument form. Write down the matrix and then column for premises and conclusion in a single row in order.

Matrix	Premise 1	Premise 2	Premise 3	Conclusion
p q	$p \bullet q$	p	q	$p \vee q$

❖ Construct a matrix for a given argument form, then assign the truth - values under each variables.

Matrix p q	Premise 1 $p \bullet q$	Premise 2 P	Premise 3 q	Conclusion $p \vee q$
T T	T T	T	T	T T
T F	T F	T	F	T F
F T	F T	F	T	F T
F F	F F	F	F	F F

- By using the truth values of propositions, assign the truth value to the premises and conclusion separately.

Matrix p q	Premise 1 p • q	Premise 2 P	Premise 3 q	Conclusion p ∨ q
T T	T **T** T	**T**	**T**	T **T** T
T F	T **F** F	**T**	**F**	T **T** F
F T	F **F** T	**F**	**T**	F **T** T
F F	F **F** F	**F**	**F**	F **F** F

Also, highlight the column under the main connective of each premise and conclusion.

❖ Next step is the criteria of deciding the validity of an argument. In the first chapter, we have learnt that in case of a valid deductive argument, if all the premises are true, its conclusion is also true. It cannot be false.

Accordingly, to determine whether the given argument form is valid, one should see all the rows in which all the premises are true. If in these rows, the conclusion is also true, then the argument is valid. **Even if in one such row where all the premises are true, and the conclusion is false. Then the argument is invalid.**

❖ We need to select those rows where premises are true. In our example, only in the first row, all the three premises are true and conclusion is also true. Therefore the given argument form is valid. The argument being substitution instance of this form is also valid.

Let's Determine the validity of some more arguments:

(1) Macro Economics and Micro Economics are sub branches of Economics.

Macro Economics is a sub branch of Economics.

Therefore, the Micro Economics is not a sub branch of Economics

(M, I)

❖ Symbolization of an argument

(1) M • I

(2) M

(3) ~ I

❖ Argument form :

(1) p • q

(2) p

∴ ~ q

Matrix p q	Premise 1 p • q	Premise 2 P	Conclusion ~ q
T T	T **T** T	**T**	**F** T
T F	T **F** F	**T**	**T** F
F T	F **F** T	**F**	**F** T
F F	F **F** F	**F**	**T** F

All the premises are true only in row no. 1 wherein the conclusion in false. Therefore the given argument is substitution instance of this argument form and therefore the above argument is invalid.

(2) Either Nainital is a city or it is a beautiful hill station.

Nainital is not a city.

Therefore it is a beautiful hill station

(C, H)

❖ Symbolization of an argument

(1) C ∨ H

(2) ~ C

∴ H

❖ Argument form :

(1) p ∨ q

(2) ~ p

∴ q

Matrix p q	Premise 1 p ∨ q	Premise 2 ~ P	Conclusion q
T T	T **T** T	**F** T	**T**
T F	T **T** F	**F** T	**F**
F T	F **T** T	**T** F	**T**
F F	F **F** F	**T** F	**F**

All the premises are true only in row no. 3 wherein even the conclusion is 'true'. Therefore the argument form is valid. The given argument is substitution instance of this argument form and therefore the above argument is valid.

(3) If either mobile games are helpful in development of personality or in achieving knowledge then it is useful in securing jobs.

Mobile games are **neither** helpful in development of personality **nor** in achieving knowledge.

Therefore mobile games are **not** useful in securing job.

(P, K, J)

❖ Symbolization of an argument

(1) (P ∨ K) ⊃ J

(2) ~ P • ~ K

∴ ~ J

❖ Argument from :

(1) (p ∨ q) ⊃ r

(2) p • ~ q

∴ ~ r

Matrix p q r	Premise 1 (p ∨ q) ⊃ r	Premise 2 ~ p • ~ q	Conclusion ~ r
T T T	T T T **T** T	F T **F** F T	**F** T
T T F	T T T **F** F	F T **F** F T	**T** F
T F T	T T F **T** T	F T **F** T F	**F** T
T F F	T T F **T** F	F T **F** T F	**T** F
F T T	F T T **T** T	T F **F** F T	**F** T
F T F	F T T **F** F	T F **F** F T	**T** F
F F T	F F F **T** T	T F **T** T F	**F** T
F F F	F F F **T** F	T F **T** T F	**T** F

All the premises are true in 7th and 8th row wherein the 8th row conclusion is true. But in 7th row where the premises are true but the conclusion is false. Therefore the given argument form is invalid. The given argument is substitution instance of this argument form. Therefore the above argument is invalid.

(4) Dr. Krishnan was a teacher and a philosopher.

If Dr. Krishnan was not a politician then he was not a philosopher.

Therefore, Dr. Krishanan was not a politician. (T, P, O)

❖ Symbolization of an argument

(1) T • P

(2) ~ O ⊃ ~ P

∴ ~ O

❖ Argument form :

(1) p • q

(2) ~ r ⊃ ~ q

∴ ~ r

Matrix p q r	Premise 1 p • q	Premise 2 ~ r ⊃ ~ q	Conclusion ~ r
T T T	T **T** T	F T **T** F T	**F** T
T T F	T **T** T	T F **F** F T	**T** F
T F T	T **F** F	F T **T** T F	**F** T
T F F	T **F** F	T F **T** T F	**T** F
F T T	F **F** T	F T **T** F T	**F** T
F T F	F **F** T	T F **F** FT	**T** F
F F T	F **F** F	F T **T** T F	**F** T
F F F	F **F** F	T F **T** T F	**T** F

All the premises are true in the 1st row and the conclusion is false. Therefore the given argument form is invalid. The given argument is substitution instance of this argument form. Therefore the above argument is invalid.

Activity : 6

With the help of truth table method determine whether the following arguments are valid or invalid.

(1) If examinations are held on time then the results will not be delayed.

It is not true that examinations are not held on time.

Therefore the results will not be delayed. (E, R)

(2) If workers join the strike then the production will suffer.

Either workers do not join the strike or production will not suffer.

Production does not suffer.

Therefore workers do not join the strike. (W, P)

(3) If Hiteksha studies hard then her mother will be happy and if she joins games then her friends will be happy.

Either she studies hard or she joins games.

Therefore either her mother will be happy or her friends will not be happy. (S, M, G, F)

3.4 Truth table as decision procedure :

Truth table method is one of the effective decision procedures by which we can solve the problem of deciding whether a propositional form is tautology, contradiction or contingency. And also decides whether an argument form is valid or invalid.

It satisfies all the conditions of effective decision procedure. i.e. reliable, mechanical and finite.

Truth table method is **reliable**. It always gives correct answer. The method never fails if one follows the basic truth values of propositions, the instructions for construction of matrix and the order of constructing the rows of truth values then the method will always be correct.

Truth table method is also **mechanical**. It goes step by step in a systematic manner. It does not require any imagination or intelligence or abstract principles to solve the problem.

Truth table method is **finite**. It has a limited number of steps. There is a last step in truth table for getting the answer.

Summary

- A decision Procedure is a method which decides whether a proposition belongs to certain class.
- Truth table is a tabular way of expressing the truth values of the truth functional statements.
- Truth table method is a decision procedure which helps us to decide whether propositional form is tautology, contradictory, contingent.
- Truth table tests the validity and invalidity of arguments.
- Truth table method is an effective decision procedure as it is reliable, mechanical and finite.

Exercises

Q. 1. Fill in the blanks with suitable words given in the brackets.

(1) ………..… is a tabular way of expressing the truth value of any truth functional compound proposition.
(Truth table, Truth tree)

(2) A tautology is a truth – functional propositional form which is ………..… under all truth possibilities of its components. *(True, False)*

(3) A contradiction is a truth – functional propositional form which is ………..… under all truth possibilities of its components. *(True, False)*

(4) A …….. is a truth – functional proposition which is true under some and false under some truth possibilities of its components.
(Contradiction, Contingency)

(5) By denying a tautology, we get a ………..…

(Contingency, Contradiction)

(6) By denying a contradiction, we can get a ………..… *(Tautology, Contingency)*

(7) By denying a contingency, we can get a ………..… *(Tautology, Contingency)*

(8) p ∨ ~ p is a ………..… *(Tautology, Contradiction)*

(9) ~ (p • ~ p) is a ………..… *(Tautology, Contingency)*

(10) p • ~ p is a ………..…

(Contingency, Contradiction)

(11) The truth table method can also be used for testing the ………..… of arguments.

(Validity, Reliability)

(12) ~ (p ∨ ~ p) is a ………..…

(Tautology, Contradiction)

(13) p ∨ q is a ………..…

(Contingency, Contradiction)

Q. 2. State whether the following statement are true or false.

(1) There are many decision procedures.

(2) The truth table method is an effective decision procedure.

(3) The truth table method is mechanical.

(4) A contradiction is a truth functional propositional form which is true under all truth possibilities of its components.

(5) A contingency is a truth – functional propositional form which is true under some and false under some truth possibilities of its components.

(6) A tautology is a truth – functional propositional form which is true under all the truth – possibilities of its components.

(7) The method of truth table requires use of intelligence.

(8) In the truth – table method, the matrix is written on the left hand side.

(9) Propositional form contains propositional variables.

(10) The method of truth table can be used to test the validity of all types of arguments.

Q. 3. Match the column.

	(A)		(B)
1.	Tautology	a.	Always false
2.	Decision procedure	b.	Sometimes true and sometimes false
3.	Contradiction	c.	Truth table
4.	Contingency	d.	Always true

Q. 4. Give logical terms for the following.

(1) A method of deciding whether an object belongs to a certain class.

(2) A tabular way of expressing the truth value of expressions containing propositional connectives.

(3) A column consists of all possible combinations of the truth values of the propositional variables in the truth function or argument.

(4) A truth functional statement form which is 'True' under all truth possibilities of its components.

(5) A truth functional statement form which is 'False' under all truth-possibilities of its components.

(6) A truth-functional statement form which is 'True' as well as 'False' under some truth possibilities of its components.

Q. 5. Give reasons for the following :

(1) Truth table is an effective decision procedure.

(2) By denying tautology, we get contradiction.

(3) By denying contradiction, we get tautology.

(4) By denying contingency, we get contingency.

Q. 6. Explain the following :

(1) Decision procedure

(2) Tautology

(3) Contradiction

(4) Contingency

(5) Truth table method as an effective decision procedure

Q. 7. Answer the following questions :

(1) What is decision procedure? What are the conditions of an effective decision procedure?

(2) Differentiate between statement form and argument form.

(3) What is truth table? How to construct a truth table?

(4) Differentiate between tautology and contradiction.

(5) How do we determine the number of rows in the truth table?

(6) Differentiate between contradiction and continguency.

(7) Why is truth table method called mechanical?

(8) Differentiate between tautology and contingency.

Q. 8. Construct truth – table to determine whether the following statement forms are tautologous, contradictory or contingent

(1) $p \bullet \sim p$

(2) $p \supset (q \supset p)$

(3) $P \vee (r \bullet p)$

(4) $(r \vee q) \equiv r$

(5) $(\sim t \bullet q) \supset (q \supset t)$

(6) $(p \supset \sim p) \bullet (\sim p \supset q)$

(7) $p \supset (p \vee r)$

(8) $\sim q \supset (q \bullet q)$

(9) $(t \supset t) \bullet (t \supset \sim t)$

(10) $[(p \supset s) \bullet p] \supset s$

(11) $[q \vee (p \bullet \sim q)] \equiv [\sim p \bullet (q \vee p)]$

(12) $(p \supset t) \bullet \sim (\sim p \vee p)$

(13) $(\sim p \bullet p) \supset [(s \vee \sim p) \bullet (\sim s \vee \sim p)]$

(14) $(p \bullet p) \vee \sim p$

(15) $\sim \{ \sim p \supset [(p \bullet q) \vee p] \}$

(16) $\sim (p \vee q) \bullet \sim (\sim p \bullet \sim q)$

(17) $[(p \bullet (q \bullet r)] \equiv [(p \bullet q) \bullet r]$

(18) $[(p \vee q) \bullet \sim p] \supset q$

(19) $(t \equiv \sim q) \supset (\sim q \supset t)$

(20) $[p \supset (r \bullet q)] \equiv [(p \supset q) \bullet (p \supset r)]$

Q. 9. With the help of truth table method, test the validity of the following arguments:

(1) $\sim M \supset N$

$\sim N$

$\therefore M \bullet N$

(2) $(P \vee Q) \bullet p$

$\therefore P$

(3) $P \supset (Q \bullet R)$

$\sim Q \vee \sim R$

$\therefore \sim P$

(4) $Q \supset p$

$\sim P$

$\therefore Q$

(5) $(P \bullet Q) \supset R$

$\sim R$

$\therefore Q$

(6) $(\sim P \vee Q) \supset P$

$P \supset R$

$\therefore (P \supset Q) \supset R$

(7) $\sim Q \vee P$

$\therefore P \supset Q$

(8) $(P \equiv Q) \supset R$

R

$\therefore \sim P \vee Q$

(9) $\sim Q \equiv S$

$P \equiv Q$

$\therefore Q \vee \sim P$

(10) $\sim (A \bullet B)$

$\sim B$

$\therefore A$

(11) $J \vee K$

$\sim J$

$\therefore \sim K$

(12) $M \supset \sim B$

$\sim B \vee M$

$\therefore B \bullet M$

(13) $\sim E \bullet M$

$\sim (M \equiv E)$

$\therefore \sim M$

(14) $C \supset F$

$\sim F \bullet C$

$\therefore \sim C$

(15) $G \equiv W$

$\sim W$

$\sim G$

$\therefore W \supset G$

Q. 10. Symbolize the following arguments using propositional constants given in the bracket and state whether they are valid or invalid by using the method of truth table.

(1) Either Germans are disciplined or they are not progressive. Germans are disciplined. Therefore they are not progressive.

(D, P)

(2) Nitin Shankar is a rhythm aranger. Therefore it is false that Nitin Shankar is both a rhythm arranger and a singer.

(R, S)

(3) If Picasso was not an Italian artist, then he was an explorer. Picasso was not an explorer. Therefore, Picasso was either an Italian artist or a dancer.

(A, E, D)

(4) It is not the case that Kalansh is serious and humorous. Kalansh is humorous. Therefore he is not serious.

(S, H)

(5) It is false that if Sparsh opts for mathematics, then he cannot offer history. Sparsh does not opt history. Therefore he opts for mathematics, but not history.

(M, H)

(6) Durvansh plays vollyball. Hence Durvansh plays vollyball but not football.

(V, F)

(7) If a man overeats, then he either invites diabetes or develops heart problems.

Some men have both diabetes and heart problems. Therefore, some men overeat.

(O, D, H)

(8) If Zoey has a strong will- power, then she can achieve many things. Zoey has a strong will power. Therefore she can achieve many things.

(W, A)

(9) Riddhi took either taxi or a bus.

If she takes a taxi, then she would be on time.

She was not on time. Therefore Riddhi took a bus.

(T, B, M)

(10) If the family planning program is effective then population can be cotrolled. The family planning program is not effective. Therefore, population cannot be controlled.

(F, P)

(11) If Het is a batsman, then Smit is a bowler. Smit is not a bowler. Hence Het is a batsman.

(B, O)

(12) Either the books are interesting or informative. If the books are informative, then they improve one's knowledge. Therefore, if the books are interesting then it improves one's knowledge.

(I, F, K)

(13) Either luck or courage is needed for success. He does not have courage. Therefore he has luck.

(L, C)

(14) If it rains, then the crops will be good. The crops are good. Therefore, it rains.

(R, C)

(15) If and only if Mann is a government servant, then he is called a public servant. Mann is not a government servant. Therefore he is not a public servant.

(G, P)

(16) Shruti loves her brothers, if and only if they work for her.

If Vinayak and Vaibhav are Shruti's brothers then they work for her.

Thererfore Shruti loves her brothers.

(S, W, V, B)

Q. 11. Complete the following table

Left component	Right component	conjunction $\bullet$	disjunction $\vee$	implication $\supset$	equivalence $\equiv$
T	T				T
T	F	F			
F	T		T		
F	F			T	

❖ ❖ ❖

Chapter 4 The Method of Deduction

For as one may feel sure that a chain will hold when he is assured that each separate link is of good material and that it clasps the two neighbouring links, viz: the one preceding and the one following it, so we may be sure of the accuracy of the reasoning when the matter is good, that is to say, when nothing doubtful enters into it and when the form consists into perpetual concatenation of truths which allow no gap - Gottfried Leibniz

4.1 DEDUCTIVE PROOF

Logic aims at distinguishing between good and bad reasoning. One of the basic problems in logic, therefore is to decide whether a given argument is valid. Another important task of logicians is to find out whether the given statement form is a tautology, contradiction or contingency. Various methods are used by logicians to deal with these. The methods are of two types : (1) Decision procedure (2) Methods that are not Decision procedures.

Truth table as we have seen is a decision procedure whereas Deductive proof is another important method used in logic which is **not a decision procedure** as all the three conditions of an effective decision procedure are not satisfied by the deductive proof. The Deductive proof is reliable, finite but not mechanical as intelligence is required to use the method. Unlike decision procedure, deductive proof is used to prove the validity of arguments and not to decide whether it is valid or invalid and it is also used to prove that the statement form is a tautology and not to decide whether it is a tautology, contradiction or contingency.

The method of deductive proof consists in deducing the conclusion of an argument from its premises by a sequence of (valid) elementary arguments. These elementary arguments are known to be valid. They are substitution instances of elementary valid argument forms which are called rules of inference.

The method of deductive proof can be used to prove the validity of deductive arguments only. In a valid deductive argument the conclusion is a logical consequence of the premises i.e. in a valid deductive argument premises imply the conclusion. Therefore, if one is able to deduce the conclusion from the premises by using valid elementary arguments, the argument is proved to be valid. The proof constructed to establish the validity of an argument by deductive proof is called formal proof of validity.

Deductive proof is of three types - 1. Direct Proof 2. Conditional Proof 3. Indirect Proof. In this chapter we will study Direct proof.

Direct proof can be used only to prove validity of arguments whereas Conditionl proof and Indirect proof can be used for proving the validity of arguments as well as tautologies.

4.2 DIRECT PROOF

The method of direct proof consists in deducing the conclusion of an argument directly from its premises by a sequence of (valid) elementary arguments. This method is called diret proof because it does not involve an assumption at any step before arriving at the conclusion

Construction of formal proof of validity involves the following steps :

1. Write down the premises in order and number them.

2. Write the conclusion on the line where the last premise is written. Separate it from the premise by a slanting line as shown below :

   1. Premise
   2. Premise
   3. Premise / ∴ Conclusion

3. Deduce the conclusion from the premises by applying rules of Inference along with rule of Replacement. Before arriving at the conclusion one may have to derive some statements. These statements can be taken as additional premises for further proof. These statements are to be numbered and the justification for each statement should be written on the right side of the statement. The justification for a statement consists in stating the number of step/steps from which the statement is derived and the rule applied to derive it. It is advised to use only one rule at a time while constructing the proof.

4. Once the conclusion is derived from the premises the proof is complete and the validity of the argument is established.

4.3 RULES OF INFERENCE AND RULE OF REPLACEMENT

For constructing formal proof of validity by deductive proof, nineteen rules are used. These nineteen rules are of two types. First nine rules of Inference form one group and are different in nature from remaining ten rules which are based on the rule of Replacement. To begin with let us study the first nine rules of inference and their application.

First nine rules of Inference are elementary valid forms of argument. Any argument which is a substitution instance of such form is also valid. With the help of these valid forms of inference one can deduce the conclusion from the premises and show that it is a logical consequence of the premises.

It should be noted that these rules can be applied only to the whole statement and not to a part of the statement. The first nine rules of inference are as follows.

(1) Modus Ponens (M. P.)

This rule is based on the nature of conditional statement. In a conditional statement the antecedent implies the consequent, which means if a conditional statement is true and its antecedent is also true, its consequent must be true, it cannot be false. The **form** of the rule is as follows -

$p \supset q$
p
$\therefore\ q$

The following argument illustrates the rule :

(a) If you study Logic then your reasoning skill improves.
You study Logic.
Therefore, your reasoning skill improves.

(b) If a student is intelligent then he will pass.
The student is intelligent.
Therefore, he will pass.

Application of the rule ---

If in an argument, a conditional statement is given as one of the premises and antecedent of the same statement is also given as another premise then by applying the rule of M. P. one can validly infer the consequent of the same conditional statement.

For example ---

(1)	$B \supset M$	
(2)	B	
(3)	$M \supset A$	/ $\therefore$ A
(4)	M	1, 2, M.P.
(5)	A	3, 4, M.P.

TRY this :

(1)	$M \supset R$	
(2)	M	
(3)	$R \supset S$	
(4)	$S \supset T$	/ $\therefore$ T
(5)	________	1, 2, M.P.
(6)	S	________
(7)	________	4, 6 M.P.

(2) Modus Tollens (M.T.)

The rule of Modus Tollens is also based on the nature of conditional statement. A conditional statement is false only when the antecedent is true and the consequent is false. Therefore if a conditional statement is true and the consequent is false then the antecedent must be false. The

form of the rule is as follows -

$p \supset q$
$\sim q$
$\therefore \quad \sim p$

The following argument illustrates the rule :

If Karan is hardworking then he will get a scholarship.
Karan did not get a scholarship.
Therefore, Karan is not hardworking.

Application of the rule ---

If in an argument a conditional statement is given as one of the premises and negation of its consequent is also given then from these two premises one can infer negation of the antecedent of that conditional statement.

For example --

(1)	$M \supset \sim T$	
(2)	$S \supset T$	
(3)	M	/ $\therefore \sim S$
(4)	$\sim T$	1, 3 M.P.
(5)	$\sim S$	2, 4 M.T.

TRY this :

(1)	$R \supset T$	
(2)	$\sim T$	
(3)	$\sim R \supset K$	/ $\therefore K$
(4)	________	1, 2, M.T.
(5)	K	________

(3) Hypothetical Syllogism (H.S.)

For this rule we need two conditional statements such that, consequent of one statement is the antecedent of the other. From such two statements we can deduce a conditional statement whose antecedent is the antecedent of the first conditional statement and consequent is the consequent of the second conditional statement. The **form** of Hypothetical Syllogism is as follows -

$p \supset q$
$q \supset r$
$\therefore \quad p \supset r$

The following argument illustrates the rule :

If it rains then the harvest is good.
If the harvest is good then the farmers are happy.
Therefore, if it rains then the farmers are happy.

Application of the rule ---

(1)	$A \supset S$	
(2)	$\sim R \supset K$	
(3)	$S \supset \sim R$	/ $\therefore A \supset K$
(4)	$A \supset \sim R$	1, 3, H.S.
(5)	$A \supset K$	4, 2, H.S.

TRY this :

(1)	$K \supset R$	
(2)	$S \supset K$	
(3)	$R \supset M$	/ $\therefore S \supset M$
(4)	$S \supset R$	________
(5)	________	4, 3, H.S.

(4) Disjunctive Syllogism (D.S.)

This rule states that if a disjunctive statement is given and its first disjunct is denied then one can affirm the second disjunct in the conclusion. This rule is based on the nature of disjunctive statement. Disjunctive statement is true when at least one of the disjuncts is true. The **form** of Disjunctive syllogism is as follows-

$p \vee q$
$\sim p$
$\therefore \quad q$

The following argument illustrates the rule :

Either Nilraj will learn to play the guitar or the piano.
Nilraj did not learn to play the guitar.
Therefore, Nilraj will learn to play the piano.

Application of the rule ---

(1)	$T \supset B$	
(2)	$\sim B$	
(3)	$T \vee R$	/ $\therefore R$
(4)	$\sim T$	1, 2, M.T.
(5)	R	3, 4, D.S.

TRY this :

(1) $R \supset T$
(2) $\sim T$
(3) $R \vee \sim S$ $\quad / \therefore \sim S$
(4) ________ $\quad$ 1, 2, M.T.
(5) $\sim S$ $\quad$ ________

(5) Constructive Dilemma (C.D.)

To apply this rule we need two statements such that, one statement is a conjunction of two conditional statements and the second statement is a disjunctive statement which affirms antecedents of the conditional statements. From such two statements we can infer a disjunctive statement which affirms consequents of the conditional statements. The **form** of Constructive Dilemma is as follows -

$(p \supset q) \bullet (r \supset s)$

$p \vee r$

$\therefore \quad q \vee s$

The following argument illustrates the rule :

If you exercise then you become healthy and if you eat fast food then you become unhealthy.

Either you exercise or you eat fast food.

Therefore, either you become healthy or unhealthy.

Application of the rule ---

(1) $A \supset (J \vee K)$
(2) A
(3) $(J \supset R) \cdot (K \supset T)$ $\quad / \therefore R \vee T$
(4) $J \vee K$ $\quad$ 1, 2, M.P.
(5) $R \vee T$ $\quad$ 3, 4, C.D.

TRY this :

(1) $(A \supset B) \cdot (R \supset S)$
(2) $M \supset (A \vee R)$
(3) M
(4) $\sim B$ $\quad / \therefore S$
(5) $A \vee R$ $\quad$ ________
(6) ________ $\quad$ 1, 5, C.D.
(7) S $\quad$ ________

(6) Destructive Dilemma (D.D.)

For this rule we need two statements such that, one statement is a conjunction of two conditional statements and the second statement is a disjunctive statement which denies consequents of the conditional statements. From such two statements we can infer a disjunctive statement which denies antecedents of the conditional statements. The **form** of Destructive Dilemma is as follows ---

$(p \supset q) \cdot (r \supset s)$

$\sim q \vee \sim s$

$\therefore \quad \sim p \vee \sim r$

The following argument illustrates the rule :

If you use solar power then it reduces pollution and if you use dustbins then you keep the city clean.

Either pollution is not reduced or you do not keep the city clean.

Therefore, either you do not use solar power or you do not use dustbins.

Application of the rule ---

(1) A
(2) $A \supset \sim P$
(3) $P \vee (\sim S \vee \sim R)$
(4) $(T \supset S) \cdot (B \supset R)$ $\quad / \therefore \sim T \vee \sim B$
(5) $\sim P$ $\quad$ 2, 1, M.P.
(6) $\sim S \vee \sim R$ $\quad$ 3, 5, D.S.
(7) $\sim T \vee \sim B$ $\quad$ 4, 6, D.D.

TRY this :

(1) $M \supset \sim R$
(2) $R \vee (\sim S \vee \sim T)$
(3) M
(4) $(J \supset S) \cdot (K \supset T)$
(5) $\sim \sim J$ $\quad / \therefore \sim K$
(6) $\sim R$ $\quad$ ________
(7) ________ $\quad$ 2, 6, D.S.
(8) $\sim J \vee \sim K$ $\quad$ ________
(9) $\sim K$ $\quad$ ________

(7) Simplification (Simp.)

The rule of Simplification states that, if a conjunctive statement is given as one of the premises then one can validly infer the first conjunct. This rule is based on the nature of conjunct. This rule is based on the nature of conjunctive statement. A conjunctive statement is true only when both the conjuncts are true, therefore, from a conjunctive statement one can derive the first conjunct. The **form** of rule of Simplification is as given below ---

$p \bullet q$

$\therefore\ p$

The following argument illustrates the rule :

Ishita practices yoga and Ishita is flexible.
Therefore, Ishita practices yoga.

Application of the rule ---

(1)	$(M \supset N) \cdot (R \supset S)$	
(2)	$(M \vee R) \cdot D$	$/ \therefore N \vee S$
(3)	$M \vee R$	2, Simp.
(4)	$N \vee S$	1, 3, C.D.

TRY this :

(1)	$\sim\sim M \cdot A$	
(2)	$\sim M \vee \sim S$	
(3)	$(A \supset S) \cdot (P \supset T)$	$/ \therefore \sim A$
(4)	$\sim\sim M$	________
(5)	________	2, 4, D.S.
(6)	________	3, Simp.
(7)	$\sim A$	________

(8) Conjunction (Conj.)

The rule of Conjunction is also based on the nature of conjunctive statement. It states that if two statements are true seperately then the conjunction of these two statements is also true. Thus from two different statements, their conjunction can validly be inferred. The **form** of rule of conjunction is as given below -

p
q
$\therefore\ p \cdot q$

The following argument illustrates the rule :

Radhika loves reading.
She writes poems.
Therefore, Radhika loves reading and she writes poems.

Application of the rule ---

(1)	$F \vee T$	
(2)	$A \supset K$	
(3)	A	
(4)	$\sim F$	$/ \therefore T \cdot K$
(5)	K	2, 3, M.P.
(6)	T	1, 4, D.S.
(7)	$T \cdot K$	6, 5, Conj.

TRY this :

(1)	$S \supset T$	
(2)	$A \supset B$	
(3)	$S \vee A$	
(4)	M	$/ \therefore (T \vee B) \cdot M$
(5)	________	1, 2, Conj.
(6)	$T \vee B$	________
(7)	________	6,4, Conj.

(9) Addition (Add.)

As per the rule of Addition, from any given statement, we can infer a disjunctive statement whose first disjunct is the statement itself and the second disjunct is any other statement. This rule is based on the nature of disjunctive statement. Such type of inference is valid because a disjunctive statement is true when at least one of the disjuncts is true. So, if 'p' is true then its disjunction with any other statement irrespective of its truth value must also be true.

The **form** of the rule is as follows -

p

$\therefore\ p \vee q$

The following argument illustrates the rule :

Tejas plays football.
Therefore, Tejas plays football or Rohan plays hockey.

Application of the rule ---

(1)	S	
(2)	$(S \cdot T) \supset A$	
(3)	T	/ $\therefore A \vee K$
(4)	$S \cdot T$	1, 3, Conj.
(5)	A	2, 4, M.P.
(6)	$A \vee K$	5, Add.

TRY this :

(1)	A	
(2)	$(A \vee S) \supset \sim T$	
(3)	$T \vee \sim M$	/ $\therefore \sim M \vee \sim S$
(4)	$A \vee S$	________
(5)	________	2, 4 M.P.
(6)	$\sim M$	________
(7)	$\sim M \vee \sim S$	________

THE RULE OF REPLACEMENT

The nine rules of Inference, cannot prove the validity of all arguments.

For example, to prove the validity of the argument- $A \cdot D$ / $\therefore$ D, nine rules are not sufficient. The Rule of replacement is therefore accepted in addition to the nine rules of Inference. The rule of replacement is also called the Principle of Extensionality.

It is based on the fact that, if any compound statement is replaced by an expression which is logically equivalent to that statement, the truth value of the resulting statement is the same as that of the original statement.

When the rule of replacement is adopted as an additional rule of inference, it allows us to infer a statement from any given statement which is logically equivalent to it. This rule can be applied to the whole as well as part of a statement. Since these rules are logically equivalent statements they can be applied in both the ways i.e. left hand expression can be replaced by right hand expression and vice versa. Based on the rule of replacement, ten logical equivalences are added to the list of rules of inference and are numbered after the nine rules. They are as follows -

(10) De Morgan's Laws (De M.)

The De Morgan's Laws are as follows -

$$\sim (p \cdot q) \equiv (\sim p \vee \sim q)$$
$$\sim (p \vee q) \equiv (\sim p \cdot \sim q)$$

The first De Morgan's law is based on the nature of conjunctive statement. Conjunctive statement is false when at least one of the conjuncts is false. So, the first De Morgan's law states that, the denial of the conjunctive statement '$\sim (p \cdot q)$' is the same as saying that either 'p' is false or 'q' is false.

The following argument illustrates the rule :

The statement, 'It is not true that Niraj is hardworking and lazy' is logically equivalent to the statement - 'Either Niraj is not hardworking or Niraj is not lazy'.

The second De Morgan's law is based on the nature of disjunctive statement. Disjunctive statement is false when both the disjuncts are false. So, the second De Morgan's law states that, the denial of the disjunctive statement '$\sim (p \vee q)$' is the same as saying that 'p' is false and 'q' is false.

The following argument illustrates the rule :

The statement, 'It is false that plastic bags are either eco friendly or are degradable' is logically equivalent to the statement - 'Plastic bags are not eco friendly and are not degradable.'

Application of the rule ---

(1)	$\sim (A \vee M)$	
(2)	$\sim (S \cdot T)$	
(3)	$A \vee J$	
(4)	$\sim \sim S$	/ $\therefore \sim T \cdot J$
(5)	$\sim A \cdot \sim M$	1, De M.
(6)	$\sim S \vee \sim T$	2, De M.
(7)	$\sim T$	6, 4, D.S.
(8)	$\sim A$	5, Simp.
(9)	J	3, 8, D.S.
(10)	$\sim T \cdot J$	7, 9, Conj.

TRY this :

(1) $S \supset T$
(2) $\sim (T \vee K)$
(3) $S \vee M$ / $\therefore M \vee \sim R$
(4) ________   2, De M.
(5) $\sim T$ ________
(6) $\sim S$ ________
(7) ________   3, 6, D.S.
(8) $M \vee \sim R$ ________

(11) Commutation (Com.)

The Commutative Laws are as follows -

$(p \cdot q) \equiv (q \cdot p)$
$(p \vee q) \equiv (q \vee p)$

Commutation means changing the place of components. The first commutative law which deals with conjunctive statement states that '$p \cdot q$' is logically equivalent to '$q \cdot p$'.

Changing the place of conjuncts makes no difference to the truth value of a statement.

The following argument illustrates the rule :

The statement, 'I like to study logic and philosophy is logically equivalent to the statement' I like to study philosophy and logic.'

The second commutative law deals with disjunctive statement and allows us to change the order of disjuncts. Changing the place of disjuncts makes no difference to the truth value of a statement.

The following argument illustrates the rule :

The statement, 'Either I will use cloth bags or paper bags' is logically equivalent to the statement 'Either I will use paper bags or cloth bags.'

Application of the rule ---

(1) $\sim (A \vee K)$
(2) $T \cdot K$ / $\therefore K \cdot \sim K$
(3) $\sim A \cdot \sim K$ 1, De M.
(4) $\sim K \cdot \sim A$ 3, Com.
(5) $K \cdot T$ 2, Com.
(6) $\sim K$ 4, Simp.
(7) K 5, Simp.
(8) $K \cdot \sim K$ 7, 6, Conj.

TRY this :

(1) $\sim S \cdot T$
(2) $(T \supset R) \cdot (A \supset B)$
(3) A / $\therefore R \cdot B$
(4) ________   1, Com.
(5) T ________
(6) $T \supset R$ ________
(7) ________   6, 5, M.P.
(8) ________   2, Com.
(9) $A \supset B$ ________
(10) ________   9, 3 M.P.
(11) $R \cdot B$ ________

(12) Association (Assoc.)

The Association Laws are as follows -

$[p \cdot (q \cdot r)] \equiv [(p \cdot q) \cdot r]$

$[p \vee (q \vee r)] \equiv [(p \vee q) \vee r]$

The Associative Laws state that in case of conjunctive and disjunctive statements if there are three components joined with the same connective i.e. either by dot or by wedge, then, whichever way you group them makes no difference to their truth value.

The following argument illustrates the first rule:

The truth value of the statement, 'Rutuja is beautiful and (hardworking and successful)' remains the same even when expressed as, '(Rutuja is beautiful and hardworking) and successful.'

The following argument illustrates the second rule :

The truth value of the statement, 'Shreyas will either eat a burger or (a sandwich or a pizza) remains the same even when expressed as, '(Shreyas will either eat a burger or a sandwich) or a pizza.'

Application of the rule ---

(1)	$(S \cdot B) \cdot T$	
(2)	$A \vee (K \vee T)$	
(3)	$\sim T$	$/ \therefore S \cdot (A \vee K)$
(4)	$S \cdot (B \cdot T)$	1, Assoc.
(5)	S	4, Simp.
(6)	$(A \vee K) \vee T$	2, Assoc.
(7)	$T \vee (A \vee K)$	6, Com.
(8)	$A \vee K$	7, 3, D.S.
(9)	$S \cdot (A \vee K)$	5,8, Conj

TRY this :

(1)	$P \vee (Q \vee M)$	
(2)	$\sim (P \vee Q)$	
(3)	$S \cdot (R \cdot A)$	$/ \therefore A \cdot M$
(4)	__________	1, Assoc.
(5)	M	__________
(6)	$(S \cdot R) \cdot A$	__________
(7)	__________	6,Com.
(8)	A	__________
(9)	$A \cdot M$	__________

(13) Distribution (Dist.)

The Distributive Laws are as follows -

$$[p \cdot (q \vee r)] \equiv [(p \cdot q) \vee (p \cdot r)]$$
$$[p \vee (q \cdot r)] \equiv [(p \vee q) \cdot (p \vee r)]$$

In the first distributive law, conjunction is distributed over disjunction. If a statement is conjoined with a disjunctive statement then it is the same as saying that, either it is conjoined with the first disjunct or it is conjoined with the second disjunct.

The following argument illustrates the rule :

The statement,

'Anuja is an actor and she is either a singer or a dancer' is logically equivalent to the statement 'Either Anuja is an actor and a singer or Anuja is an actor and a dancer.'

In the second distributive law, disjunction is distributed over conjunction. If a statement is in disjunction with a conjunctive statement then it is the same as saying that, it is in disjunction with the first conjunct and it is in disjunction with the second conjunct.

The following argument illustrates the rule :

The statement,

'Either Vikas plays cricket or he sings and paints' is logically equivalent to the statement 'Either Vikas plays cricket or he sings and either Vikas plays cricket or he paints'.

Application of the rule ---

(1)	$\sim (S \bullet A)$	
(2)	$S \cdot (A \vee B)$	
(3)	$K \vee (P \cdot D)$	$/ \therefore (S \cdot B) \cdot (K \vee D)$
(4)	$(S \cdot A) \vee (S \cdot B)$	2, Dist.
(5)	$S \cdot B$	4, 1, D. S.
(6)	$(K \vee P) \cdot (K \vee D)$	3, Dist.
(7)	$(K \vee D) \cdot (K \vee P)$	6, Com.
(8)	$K \vee D$	7, Simp.
(9)	$(S \cdot B) \cdot (K \vee D)$	5, 8, Conj.

TRY this :

(1)	$P \vee (R \cdot S)$	
(2)	$\sim R$	
(3)	$\sim (P \vee M)$	$/ \therefore \sim M \cdot P$
(4)	__________	1, Dist.
(5)	$P \vee R$	__________
(6)	__________	5, Com.
(7)	P	__________
(8)	__________	3, DeM.
(9)	$\sim M \cdot \sim P$	__________
(10)	__________	9, Simp.
(11)	$\sim M \cdot P$	__________

(14) Double Negation (D. N.)

The form of this rule is as follows -

$$p \equiv \sim \sim p$$

The rule of Double Negation states that a statement is equivalent to the negation of its contradictory.

The following argument illustrates the rule :

To say that, 'Global warming is a current world crisis' is logically equivalent to saying, 'It is not the case that global warming is not a current world crisis.'

Application of the rule ---

(1)	~ R ∨ (S ∨ B)	
(2)	R	
(3)	~ S	/ ∴ ~ ~ B
(4)	~ ~ R	2, D. N.
(5)	S ∨ B	1, 4, D. S.
(6)	B	5, 3, D. S.
(7)	~ ~ B	6, D. N.

TYR this :

(1)	~ A ⊃ B	
(2)	~ B	
(3)	~(~ M ∨ R)	/ ∴ A · M
(4)	__________	1, 2, M. T.
(5)	A	__________
(6)	__________	3, DeM.
(7)	__________	6, D.N.
(8)	M	__________
(9)	A · M	__________

(15) Transposition (Trans.)

The rule of Transposition is expresssed as follows :

(p ⊃ q) ≡ (~ q ⊃ ~ p)

Like commutative laws this rule allows us to change the places of components. However, when we interchange the antecedent and consequent, we have to negate both of them so that the truth value remains the same.

The following argument illustrates the rule :

To say that, 'If people take efforts then environmental pollution can be controlled' is logically equivalent to saying that, 'If environmental pollution is not controlled then people have not taken efforts.'

Application of the rule ---

(1)	~ ~ K	
(2)	K ⊃ A	/ ∴ ~ ~ A
(3)	~ A ⊃ ~ K	2, Trans.
(4)	~ ~ A	3, 1, M. T.

TRY this :

(1)	T ⊃ A	
(2)	~ S ⊃ R	
(3)	(~ A ⊃ ~ T) ⊃ ~ R	/ ∴ S ∨ (B · Q)
(4)	~ A ⊃ ~ T	__________
(5)	__________	3,4 M.P.
(6)	~ ~ S	__________
(7)	__________	6, D.N.
(8)	S ∨ (B · Q)	__________

(16) Material Implication (Impl.)

The rule is stated as follows -

(p ⊃ q) ≡ (~ p ∨ q)

This rule is based on the nature of conditional statement. A conditional statement is false only when its antecedent is true and consequet is false. But if antecedent is false then whatever may be the truth value of consequent the conditional statement is true or if consequent is true then whatever may be the truth value of antecedent the conditional statement is true. Therefore the rule of implication states that, if 'p ⊃ q' is true then either 'p' is false or 'q' is true.

The following argument illustrates the rule :

To say that, 'If you litter on streets then you are irresponsible.' is logically equivalent to the statement, 'Either you do not litter on streets or you are irresponsible.'

Application of the rule ---

(1)	$(A \supset B) \vee S$	
(2)	A	
(3)	$\sim B$	$/ \therefore S$
(4)	$(\sim A \vee B) \vee S$	1, Impl.
(5)	$\sim A \vee (B \vee S)$	4, Assoc.
(6)	$\sim \sim A$	2, D.N.
(7)	$B \vee S$	5, 6, D.S.
(8)	S	7, 3, D.S.

TRY this :

(1)	$Q \supset T$	
(2)	$(\sim Q \vee T) \supset M$	
(3)	$T \supset S$	$/ \therefore M \cdot (\sim Q \vee S)$
(4)	$\sim Q \vee T$	________
(5)	________	2,4 M.P.
(6)	$Q \supset S$	________
(7)	________	6, Impl.
(8)	$M \cdot (\sim Q \vee S)$	________

(17) Material Equivalence - (Equiv.)

The two rules are as given below -

$$(p \equiv q) \equiv [(p \supset q) \cdot (q \supset p)]$$
$$(p \equiv q) \equiv [(p \cdot q) \vee (\sim p \cdot \sim q)]$$

The first rule states the nature of bi-conditional statement i.e. in a bi-conditional statement both the components imply each other. The truth condition of a materially equivalent statement is expressed in the second rule i.e. a materially equivalent statement is true either when both the components are true or when both are false.

The following argument illustrates this rule :

According to the first rule, the statement, 'If and only if you pursue your passion then you will succeed,' is logically equivalent to the statement, 'If you pursue your passion then you will succeed and if you succeed then you have pursued your passion.'

As per the second rule, the same statement is logically equivalent to the statement,' Either you pursue your passion and succeed or you do not pursue your passion and you do not succeed.'

Application of the rule ---

(I)

(1)	$S \equiv M$	
(2)	$\sim S$	$/ \therefore \sim M$
(3)	$(S \supset M) \cdot (M \supset S)$	1, Equiv.
(4)	$(M \supset S) \cdot (S \supset M)$	3, Com.
(5)	$M \supset S$	4, Simp.
(6)	$\sim M$	5, 2, M.T.

(II)

(1)	$S \equiv M$	
(2)	$\sim S$	$/ \therefore \sim M$
(3)	$(S \cdot M) \vee (\sim S \cdot \sim M)$	1, Equiv.
(4)	$\sim S \vee \sim M$	2, Add.
(5)	$\sim (S \cdot M)$	4, DeM.
(6)	$\sim S \cdot \sim M$	3, 5, D.S.
(7)	$\sim M \cdot \sim S$	6, Com.
(8)	$\sim M$	7, Simp.

TRY this :

(1)	$A \equiv S$	
(2)	S	
(3)	$(K \cdot T) \vee (\sim K \cdot \sim T)$	
(4)	$(K \equiv T) \supset \sim P$	
(5)	$P \vee M$	$/ \therefore M \cdot A$
(6)	$(A \supset S) \cdot (S \supset A)$	________
(7)	________	6, Com.
(8)	$S \supset A$	________
(9)	________	8, 2, M.P.
(10)	________	3, Equiv.
(11)	$\sim P$	________
(12)	________	5, 11 D.S.
(13)	$M \cdot A$	________

(18) Exportation (Exp.)

The rule is as follows -

$$[(p \cdot q) \supset r] \equiv [p \supset (q \supset r)]$$

This rule is applied when we have a conditional statement having three components.

In such a case it is the same as saying that, first and second components both imply the third one. First implying the second and second implying the third.

The following argument illustrates the rules:

'If you drink and drive then an accident can take place'is logically equivalent to the statement,'If you drink then if you drive then an accident can take place.'

Application of the rule ---

(1)	B	
(2)	$(B \cdot S) \supset T$	
(3)	$T \supset R$	/ $\therefore S \supset R$
(4)	$B \supset (S \supset T)$	2, Exp.
(5)	$S \supset T$	4, 1, M. P.
(6)	$S \supset R$	5, 3, H.S.

TRY this :

(1)	$\sim P \supset (Q \supset \sim S)$	
(2)	$\sim P \cdot Q$	/ $\therefore S \supset S$
(3)	________	1, Exp.
(4)	$\sim S$	________
(5)	________	4, Add.
(6)	$S \supset S$	________

(19) Tautology (Taut.)

The rule is as follows-

$p \equiv (p \cdot p)$
$p \equiv (p \vee p)$

This rule states that any statement is equivalent to an expression where the statement is in conjunction with itself or the statement is in disjunction with the statement itself.

The following argument illustrates the rule:

According to the first rule, the statement, 'The weather is pleasant' is logically equivalent to the statement,' The weather is pleasant and the weather is pleasant' and as per the second rule, the statement, 'The weather is pleasant' is logically equivalent to the statement, 'The weather is pleasant or the weather is pleasant.'

Application of the rule ---

(1)	$(S \supset R) \cdot (B \supset R)$	
(2)	$(\sim K \cdot \sim K) \supset M$	
(3)	$\sim M$	
(4)	$S \vee B$	/ $\therefore R \cdot K$
(5)	$R \vee R$	1, 4, C. D.
(6)	R	5, Taut.
(7)	$\sim K \supset M$	2, Taut.
(8)	$\sim \sim K$	7, 3, M. T.
(9)	K	8, D. N.
(10)	$R \cdot K$	6, 9, Conj.

TRY this :

(1)	$(A \supset B) \bullet (M \supset N)$	
(2)	$\sim B \vee \sim B$	
(3)	$A \vee M$	
(4)	$(\sim N \vee S) \vee (\sim N \vee S)$	/ $\therefore \sim S \supset \sim R$
(5)	________	1, 3 C.D.
(6)	$\sim B$	________
(7)	________	5, 6, D.S.
(8)	________	4, Taut.
(9)	$\sim \sim N$	________
(10)	________	8, 9, D.S.
(11)	$S \vee \sim R$	________
(12)	________	11, Impl.

Rules of Inference :

(1) Modus Ponens (M.P.)

$p \supset q$

p

$\therefore$ q

(2) Modus Tollens (M. T.)

$p \supset q$

$\sim q$

$\therefore$ $\sim p$

(3) Hypothetical Syllogism (H. S.)

$p \supset q$

$q \supset r$

$\therefore$ $p \supset r$

(4) Disjunctive Syllogism (D. S.)

$p \vee q$

$\sim p$

$\therefore$ q

(5) Construtive Dilemma (C. D.)

$(p \supset q) \cdot (r \supset s)$

$p \vee r$

$\therefore$ $q \vee s$

(6) Destrutive Dilemma (D.D.)

$(p \supset q) \cdot (r \supset s)$

$\sim q \vee \sim s$

$\therefore$ $\sim p \vee \sim r$

(7) Simplification (Simp.)

$p \cdot q$

$\therefore$ p

(8) Conjunction (Conj.)

p

q

$\therefore$ $p \cdot q$

(9) Addition (Add.)

p

$\therefore$ $p \vee q$

Rule of Replacement :

(10) De Morgan's Laws (De M.)

$\sim (p \cdot q) \equiv (\sim p \vee \sim q)$

$\sim (p \vee q) \equiv (\sim p \cdot \sim q)$

(11) Commutation (Com.)

$(p \cdot q) \equiv (q \cdot p)$

$(p \vee q) \equiv (q \vee p)$

(12) Association (Assoc.)

$[p \cdot (q \cdot r)] \equiv [(p \cdot q) \cdot r]$

$[p \vee (q \;\; r)] \equiv [(p \vee q) \vee r]$

(13) Distribution Laws (Dist.)

$[p \cdot (q \vee r)] \equiv [(p \cdot q) \vee (p \cdot r)]$

$[p \vee (q \cdot r)] \equiv [(p \vee q) \cdot (p \vee r)]$

(14) Double Negation (D.N.)

$p \equiv \sim \sim p$

(15) Transposition (Trans.)

$(p \supset q) \equiv (\sim q \supset \sim p)$

(16) Material Implication - (Impl.)

$(p \supset q) \equiv (\sim p \vee q)$

(17) Material Equivalence - (Equiv.)

$(p \equiv q) \equiv [(p \supset q) \cdot (q \supset p)]$

$(p \equiv q) \equiv [(p \cdot q) \vee (\sim p \cdot \sim q)]$

(18) Exportation (Exp.)

$[(p \cdot q) \supset r] \equiv [p \supset (q \supset r)]$

(19) Tautology (Taut.)

$p \equiv (p \cdot p)$

$p \equiv (p \vee p)$

Summary

- The method of deductive proof is used for proving the validity of arguments. It consists in deducing the conclusion of an argument from its premises by a sequence of valid elementary arguments.
- The method of deductive proof is not a decision procedure, as it is not mechanical.
- The method of direct proof consists in deducing the conclusion of an argument directly from its premises by a sequence of (valid) elementary arguments.
- In the method of deductive proof, nineteen rules are used for constructing formal proof of validity.
- The first nine rules of inference are elementary valid forms of arguments. Remaining ten rules are logically equivalent statements, based on the rule of replacement.
- Rules of inference can be applied only to the whole statement. Rules based on the rule of Replacement can be applied to the whole as well as part of the statement.

Exercises

Q. 1. Fill in the blanks with suitable words from those given in the brackets :

1. According to De Morgan's Law (DeM.), ~ (S • ~R) ≡ ....

   [(S ∨ R) / (~ S ∨ ~ ~ R)]

2. The rule involved in (A ∨ M) ≡ (M ∨ A) is .... *[Commutation / Transposition]*

3. Thr rule of Simplification (Simp.) is based on the nature of .... statement. *[Disjunctive / Conjunctive]*

4. (B ⊃ ~ R) ≡ .... is by the rule of Material Implication (Impl.) *[(~ B ∨ ~ R) / (B ∨ R)]*

5. The rule used in ~ T ≡ (~ T ∨ ~ T) is .... *[Tautology / Commutation]*

6. [p • (q • r)] ≡ [(p • q) • r] is by the rule of .... *[Association / Exportation]*

7. (K ⊃ T) ≡ .... is by the rule of Transposition (Transp.)

   [(T ⊃ ~ K) / (~ T ⊃ ~ K)]

8. The rule of Modus Tollens is based on the nature of .... statement. *[Conjunctive / Conditional]*

9. [(p • q) ⊃ r] ≡ [p ⊃ (q ⊃ r)] is by the rule of .... *[Distribution / Exportation]*

10. The rule of replacement can be applied to .... of the statement. *[Whole / Whole as well as part]*

Q. 2. State whether the following statements are True or False :

1. Rules of inference can be applied to the part of the statement.
2. The method of deductive proof is a decision procedure.
3. The rule of Disjunctive Syllogism (D.S.) can be applied to the part of the statement.
4. The method of direct proof consists in deducing the conclusion directly from the premises.
5. p /∴ p ∨ q is the rule of simplification (Simp.)

6. $[(p \supset q) \cdot p] \supset q$ is the rule of Modus Ponens (M.P.)
7. In the rule of Transposition (Trans.), places of antecedent and consequent are changed and both of them are negated.
8. The method of deductive proof is a mechanical method.
9. The rule of Hypothetical syllogism (H.S.) is based on the nature of disjunctive statement.
10. $p, q \,/\, \therefore p \cdot q$ is the rule of Addition (Add.)

Q. 3. Match the columns :

	(A)		(B)
1.	p	1.	$(\sim p \vee q)$
2.	$(p \supset q)$	2.	$(\sim p \vee \sim q)$
3.	$(p \equiv q)$	3.	$[(p \vee q) \cdot (p \vee r)]$
4.	$\sim (p \cdot q)$	4.	$\sim \sim p$
5.	$[p \vee (q \cdot r)]$	5.	$[(p \supset q) \cdot (q \supset p)]$

Q. 4. Give reasons for the following :

1. The method of deductive proof is not a decision procedure.
2. The nine rules of inference can be applied to the whole statement only.
3. The rules based on the rule of replacement can be applied to the whole as well as part of the statement.

Q. 5. Explain the following :

1. Rule of Association.
2. Rule of Distribution.
3. Rule of Constructive Dilemma
4. Rule of Destructive Dilemma.
5. Rule of Addition.
6. Rule on De Morgan's Laws.
7. Rule of Double Negation.
8. Rule of Material Implication.
9. Rule of Material Equivalence.
10. Rule of Exportation.
11. Rule of Tautology.

Q. 6. Answer the following questions :

1. Explain the method of Deductive proof.
2. Explain the method of Direct Deductive proof.
3. Distinguish between rules of Inference and Rule of Replacement.
4. Distinguish between rule of Modus Ponens and rule of Modus Tollens.
5. Distinguish between rule of Hypothetical Syllogism and rule of Disjunctive Syllogism.
6. Distinguish between rule of Simplification and rule of Conjunction.
7. Distinguish between rule of Commutation and rule of Transposition.

Q. 7. State whether the following arguments are valid or invalid :

(1) $(A \supset B) \supset \sim C$
$A \supset B$
$\therefore C$

(2) $(M \cdot N) \vee (T \equiv S)$
$M \cdot N$
$\therefore T \equiv S$

(3) $L \supset (K \vee L)$
$\sim L$
$\therefore K \vee L$

(4) $\sim R \supset (T \cdot W)$
$\sim (T \cdot W)$
$\therefore R$

(5) $(S \supset \sim T) \cdot (R \supset W)$
$S \vee R$
$\therefore \sim T \vee W$

(6) $(H \supset L) \cdot (K \supset J)$
$\sim L \vee \sim J$
$\therefore \sim H \vee \sim K$

(7) $(R \equiv S) \cdot (M \supset N)$
$R \vee M$
$\therefore S \vee N$

(8) $(T \supset W) \bullet L$

$\therefore T \supset W$

(9) $S \lor \sim L$

$\sim T \supset W$

$\therefore (S \lor \sim L) \bullet (\sim T \supset W)$

(10) $J \supset L$

$\sim L \supset K$

$\therefore J \supset K$

Q. 8. State whether the following equivalances are correct or incorrect :

(1) $\sim (p \lor \sim q) \equiv (\sim p \bullet q)$

(2) $\sim \sim R \equiv R$

(3) $(\sim K \lor \sim K) \equiv K$

(4) $[(R \bullet \sim S) \bullet \sim T] \equiv [R \lor (\sim S \lor \sim T)]$

(5) $[\sim A \bullet (B \lor C)] \equiv [(\sim A \bullet B) \lor (\sim A \bullet C)]$

(6) $(\sim p \supset \sim q) \equiv (q \supset p)$

(7) $(\sim S \bullet \sim T) \equiv (T \bullet S)$

(8) $(\sim p \supset q) \equiv (p \lor q)$

(9) $[(p \bullet q) \lor (q \bullet p)] \equiv (p \equiv q)$

(10) $[(p \supset q) \supset r] \equiv [p \bullet (q \supset r)]$

Q. 9. State the justification for each step of the following arguments :

(1) 1 $(K \lor S) \cdot (K \lor \sim T)$

2 $S \supset T$ / $\therefore K$

3 $K \lor (S \cdot \sim T)$

4 $\sim S \lor \sim \sim T$

5 $\sim S \lor T$

6 $\sim (S \cdot \sim T)$

7 $(S \cdot \sim T) \lor K$

8 K

(2) 1 $(W \supset L) \cdot (W \supset K)$

2 $(L \cdot K) \supset Z$

3 $\sim Z$ / $\therefore \sim W$

4 $\sim (L \cdot K)$

5 $\sim L \lor \sim K$

6 $\sim W \lor \sim W$

7 $\sim W$

(3) 1 $(X \supset \sim Y) \cdot (Z \supset A)$

2 $\sim (\sim X \cdot \sim Z)$ / $\therefore Y \supset A$

3 $\sim \sim X \lor \sim \sim Z$

4 $X \lor Z$

5 $\sim Y \lor A$

6 $Y \supset A$

(4) 1 $(A \lor B) \supset \sim C$

2 C / $\therefore \sim B$

3 $\sim \sim C$

4 $\sim (A \lor B)$

5 $\sim A \cdot \sim B$

6 $\sim B \cdot \sim A$

7 $\sim B$

(5) 1 $\sim L \supset K$

2 $(L \lor M) \supset (U \cdot W)$

3 $\sim K$ / $\therefore U \lor U$

4 $\sim \sim L$

5 L

6 $L \lor M$

7 $U \cdot W$

8 U

9 $U \lor U$

(6) 1 $W \lor S$

2 $\sim S$

3 $(W \cdot X) \supset Y$ / $\therefore \sim X \lor Y$

4 $S \lor W$

5 W

6 $W \supset (X \supset Y)$

7 $X \supset Y$

8 $\sim X \lor Y$

(7) 1 $(A \cdot B) \cdot C$

2 $A \supset (D \lor K)$

3 $\sim D$ / $\therefore K$

4 $A \cdot (B \cdot C)$

5 A

6 $D \lor K$

7 K

(8) 1 $K \vee L$
2 $(L \cdot M) \supset (O \cdot P)$
3 $\sim K$
4 M / $\therefore G \supset O$
5 L
6 $L \cdot M$
7 $O \cdot P$
8 O
9 $O \vee \sim G$
10 $\sim G \vee O$
11 $G \supset O$

(9) 1 $\sim D \vee E$
2 $E \supset G$
3 $(\sim G \supset \sim D) \supset H$ / $\therefore H \vee K$
4 $D \supset E$
5 $D \supset G$
6 $\sim G \supset \sim D$
7 H
8 $H \vee K$

(10) 1 $A \supset B$
2 $C \supset D$
3 $\sim (B \cdot D)$ / $\therefore \sim A \vee \sim C$
4 $(A \supset B) \cdot (C \supset D)$
5 $\sim B \vee \sim D$
6 $\sim A \vee \sim C$

Q. 10. Construct formal proof of validity for the following arguments using nine rules of Inference :

(1) 1 $P \supset Q$
2 $P \supset R$
3 P / $\therefore Q \cdot R$

(2) 1 $T \supset P$
2 $\sim P$
3 $\sim T \supset \sim R$ / $\therefore \sim R \vee S$

(3) 1 $M \supset N$
2 $N \supset O$
3 $(M \supset O) \supset (N \cdot P)$ / $\therefore N \vee R$

(4) 1 $A \vee B$
2 $\sim A$
3 $M \cdot D$ / $\therefore B \cdot M$

(5) 1 $M \vee \sim S$
2 $\sim M$
3 $P \supset S$ / $\therefore \sim P \vee R$

(6) 1 $\sim A$
2 $\sim B$
3 $(\sim A \cdot \sim B) \supset R$ / $\therefore R$

(7) 1 $A \cdot S$
2 $A \supset \sim B$
3 $B \vee T$ / $\therefore T \vee \sim M$

(8) 1 $W \vee T$
2 $(W \vee T) \supset (L \cdot \sim S)$ / $\therefore L$

(9) 1$(P \supset Q) \cdot R$
2 $(Q \supset R) \cdot S$ / $\therefore P \supset R$

(10) 1 $(A \cdot B) \supset S$
2 $S \supset R$
3 A
4 B / $\therefore R$

(11) 1 $(T \vee S) \supset P$
2 $P \supset Q$
3 T / $\therefore Q$

(12) 1 $Q \supset S$
2 $P \supset T$
3 $Q \vee P$
4 $\sim S$ / $\therefore T$

(13) 1 $(M \vee O) \supset (A \cdot M)$
2 $(A \cdot M) \supset (D \cdot E)$
3 M / $\therefore D$

(14) 1 $P \supset T$
2 $T \supset \sim D$
3 $\sim D \supset M$ / $\therefore P \supset M$

(15) 1 $H \supset K$
2 $T \vee F$
3 H
4 $\sim T$ / $\therefore F \cdot K$

(16) 1 $A \supset (B \vee S)$

2 $\sim (B \vee S)$

3 $D \supset L$

4 $A \vee D$ / $\therefore L$

(17) 1 $A \vee B$

2 $B \supset M$

3 $A \supset D$

4 $\sim D$ / $\therefore B \cdot (A \vee B)$

(18) 1 $A \supset B$

2 $\sim A \supset \sim C$

3 $C \vee (D \cdot E)$

4 $\sim B$ / $\therefore D \vee (S \equiv \sim R)$

(19) 1 $\sim S \supset (P \supset T)$

2 $\sim (P \supset T)$

3 $A \supset M$

4 $\sim S \vee A$ / $\therefore M \vee (R \cdot Q)$

(20) 1 $\sim S \cdot (A \vee B)$

2 $(M \supset S) \cdot R$

3 $M \vee \sim T$ / $\therefore \sim T \vee \sim K$

(21) 1 $A \supset M$

2 $P \supset T$

3 $P \vee A$

4 $\sim T$ / $\therefore M$

(22) 1 $S \supset M$

2 $P \supset A$

3 $\sim A \vee \sim M$

4 $K \cdot S$ / $\therefore (\sim P \vee \sim S) \cdot K$

(23) 1 $R \supset S$

2 $A \supset B$

3 $\sim T$

4 $\sim S \vee \sim B$ / $\therefore (\sim R \vee \sim A) \cdot \sim T$

(24) 1 $A \supset (\sim B \vee \sim D)$

2 $D \supset A$

3 D

4 $A \supset B$

5 $M \supset D$ / $\therefore \sim A \vee \sim M$

(25) 1 $R \supset T$

2 $S \supset B$

3 $R \cdot M$

4 $\sim T$ / $\therefore B \vee \sim A$

(26) 1 $R \vee S$

2 $[(R \vee S) \vee K] \supset \sim L$

3 T / $\therefore \sim L \cdot T$

(27) 1 $\sim K \cdot \sim S$

2 $M \vee T$

3 $M \supset K$ / $\therefore T \vee (S \supset R)$

(28) 1 $\sim A \supset R$

2 $S \supset \sim A$

3 $\sim R$

4 $S \vee \sim P$ / $\therefore \sim P$

(29) 1 $L \vee \sim S$

2 $\sim A$

3 $(\sim A \vee \sim M) \supset \sim L$

4 $P \cdot B$ / $\therefore \sim S \cdot (P \cdot B)$

(30) 1 $A \supset \sim B$

2 $A \cdot \sim R$

3 $B \vee (S \vee \sim M)$

4 $\sim S \cdot \sim T$ / $\therefore A \cdot \sim M$

Q.11. Construct formal proof of validity for the following arguments using the rule of Inference and Replacement :

(1) 1 $\sim (M \cdot R)$

2 M

3 $(\sim R \supset B) \cdot (A \supset K)$ / $\therefore B \vee K$

(2) 1 $B \cdot A$

2 $\sim A \vee S$

3 $S \supset T$ / $\therefore T \vee (\sim R \supset M)$

(3) 1 $A \vee (B \vee M)$

2 $\sim B$ / $\therefore A \vee M$

(4) 1 $M \supset N$

2 $A \supset N$

3 $M \vee A$ / $\therefore N$

(5) 1 $R \vee (S \cdot T)$
2 $\sim T$
3 $\sim S$ $\quad / \therefore R$

(6) 1 $\sim (S \vee T)$
2 $\sim S \supset \sim P$
3 $P \vee R$ $\quad / \therefore R \vee \sim M$

(7) 1 $A \supset \sim B$
2 $A \cdot S$
3 $B \vee R$ $\quad / \therefore R \cdot S$

(8) 1 $T \supset \sim S$
2 $T \vee T$
3 $S \vee \sim K$ $\quad / \therefore \sim K \vee \sim K$

(9) 1 $\sim K \supset \sim T$
2 $\sim K \cdot S$
3 $\sim T \supset R$
4 $(R \cdot S) \supset M$ $\quad / \therefore M \vee M$

(10) 1 $S \supset T$
2 $T \supset M$ $\quad / \therefore M \vee \sim S$

(11) 1 $A \supset M$
2 $(\sim A \vee M) \supset R$
3 $\sim S \vee T$ $\quad / \therefore (S \supset T) \cdot R$

(12) 1 $A \supset (B \supset M)$
2 $A \cdot B$ $\quad / \therefore M \cdot [(A \cdot B) \supset M]$

(13) 1 $P \equiv S$
2 $\sim P$ $\quad / \therefore \sim S \vee \sim M$

(14) 1 $A \vee (R \vee \sim P)$
2 P $\quad / \therefore A \vee R$

(15) 1 $W \vee B$
2 $W \supset \sim S$
3 $B \supset \sim S$
4 $T \supset S$ $\quad / \therefore \sim T$

(16) 1 $\sim B \vee M$
2 $M \supset R$ $\quad / \therefore \sim R \supset \sim B$

(17) 1 $(S \cdot T) \supset P$
2 $P \supset F$
3 $\sim F$ $\quad / \therefore \sim S \vee \sim T$

(18) 1 $(R \supset Q) \cdot (Q \supset R)$
2 $(B \vee M) \vee S$
3 $\sim B$
4 $\sim S$ $\quad / \therefore (R \equiv Q) \cdot M$

(19) 1 $\sim (S \vee M)$
2 $P \supset M$
3 $M \vee \sim N$ $\quad / \therefore \sim (P \vee N)$

(20) 1 $S \vee T$
2 $(S \vee M) \supset (Q \cdot B)$
3 $\sim B$ $\quad / \therefore T$

(21) 1 $\sim (\sim A \vee R)$
2 R $\quad / \therefore T \cdot A$

(22) 1 $(R \cdot M) \supset S$
2 R $\quad / \therefore \sim S \supset \sim M$

(23) 1 $(S \cdot T) \vee (\sim S \cdot \sim T)$
2 $\sim S \vee \sim R$
3 $\sim (\sim S \cdot \sim T)$ $\quad / \therefore \sim (R \cdot B) \cdot (S \equiv T)$

(24) 1 $\sim A \vee B$
2 $S \supset T$
3 $A \vee S$ $\quad / \therefore \sim B \supset T$

(25) 1 $\sim (A \vee M)$
2 $S \supset A$
3 $M \vee \sim R$ $\quad / \therefore \sim (S \vee R)$

(26) 1 $R \vee (S \cdot T)$
2 $(R \vee T) \supset \sim M$ $\quad / \therefore M \supset F$

(27) 1 $S \supset A$
2 $B \supset S$
3 $\sim T \cdot \sim A$ $\quad / \therefore \sim B \cdot \sim T$

(28) 1 $S \supset T$
2 $R \vee S$ $\quad / \therefore \sim T \supset R$

(29) 1 $(R \supset S) \cdot (R \supset M)$
2 $\sim S \vee \sim M$ $\quad / \therefore \sim (T \cdot R)$

(30) 1 $B \supset K$
2 $\sim B \supset S$ $\quad / \therefore (K \vee S) \vee \sim A$

Chapter 5 Inductive Inference and its Types

*When general observations are drawn from so many particulars as to become certain and indubitable, these are jewels of knowledge - **Samuel Johnson***

DO YOU KNOW THAT ....

Many scientific discoveries and inventions are the results of inductive reasoning.

When you are wearing the same brand you are using analogy.

Many a times women use inductive reasoning while cooking food.

Knowingly or unknowingly we all make use of inductive reasoning in our day to day life.

Need for induction :

In the previous chapters we have dealt with the formal aspect of logic i.e. deductive logic. Deductive logic determines the relations between premise/s and conclusion without considering its content matter. Deduction is concerned with the form and not with the content of an argument. Conclusion of the deductive inference is certain but it does not gives us any new information or knowledge whereas the conclusion of inductive inference is always probable but it does gives us new information or knowledge and hence there is a need for induction.

INDUCTIVE INFERENCE

The aim of inductive inference is to establish the material truth. In inductive inference the conclusion asserts something more than what is given in the premises, for example : when we say that -

Gold expands on heating.

Silver expands on heating.

Iron expands on heating.

∴ All metals expand on heating.

In the above example on the basis of our observation of some metals expanding on heating, we make a generalization about all metals expanding on heating.

Inductive Inference is not only used for establishing general propositions but also particular propositions. Inductive inferences are of four main kinds. They are :

1. Simple enumeration
2. Analogy
3. Scientific induction
4. Hypothetico-deductive method

Out of these four, the first and the third type of inductive inference establish general propositions. The second one i.e. analogy establishes a conclusion which is about a particular proposition and the last one i.e. hypothetico-deductive method may be used to establish a general proposition or a particular proposition.

*** Activity :** state which of the following inferences are deductive inference and which are inductive inference.

1. All those who can afford medical insurance are employed.
   All actors can afford medical insurance.
   ∴ All actors are employed.

2. Sunita bought an apartment in the same building as Latika's. She paid the same price, the carpet area of her apartment is the same as Latika's apartment. Latika's apartment has five bedrooms.
   ∴ Sunita's apartment must also have five bedrooms.

3. Whoever exists is a human being.
   Pen exists.
   ∴ Pen is a human being.

4. Everytime I organized a house party, my friend comes late. Today I have organized a house party so I am sure that my friend will come late.

Simple Enumeration

Simple Enumeration is a common man's method of arriving at a generalization. Generalization is a statement of the type, 'All A is B'. It is the simplest kind of induction. The generalization of a common man differs from that of a scientist. Common man uses simple enumeration whereas scientist use scientific induction for establishing generalizations. Simple enumeration is the process of establishing a generalization on the basis of the observation of some cases or instances of a kind. Generalization in simple enumeration is supported by direct evidence. In induction by simple enumeration we generalize by going beyond what has been experienced. Induction by simple enumeration can be defined as **"what is true of several cases of a kind is true of all the cases of that kind".** It establishes a generalization on the basis of uniform and uncontradicted experience. For example :

First observed crow is black.

Second observed crow is black.

Third observed crow is black.

One lakh observed crows are black.

∴ All crows are black.

A Few more examples of simple enumeration are :

(1) Some roses have thorns.
∴ All roses have thorms.

(2) Some observed flowers have fragrance.
∴ All flowers have fragrance.

The form of simple enumeration is as follows :

All observed P's are Q.
No observed P is non Q.
∴ All P's are Q.

Generalizations established by simple enumeration have the following characteristics.

- **Uniform and Uncontradicted Experience :**

Generalization in simple enumeration is based on uniform and uncontradicted expeience.

For example : Ice is cold, fire is hot etc. We have never come across any contrardictory experience of ice being hot and fire being cold. In these examples the scope of generalizations are unlimited hence it is larger than the scope of evidence.

- **Absence / Lack of analysis of property:** Simple enumeration is the process of simply counting the instances (cases) to find that all these cases share a common property, However it does not involve analysis : for example - why crows are black, or why roses have thorns. Here one is not concerned in finding out why blackness goes with crows or why thorns are associated with roses.

- **Unrestricted generality :**

The generalization established by simple enumeration is not about a class with limited number of members, for example :

Some students in this class are smart

∴ All students in this class are smart.

In the given example a generalization is established but it is of restricted generality. Therefore such kind of arguments are not induction by simple enumeration. In Simple Enumeration the conclusion i.e. generalization is about unrestricted number of members. For example :

Some polar bears are white
∴ All polar bears are white

In case of simple Enumeration there is an inductive leap or jump from observed cases to unobserved or known cases to unknown cases. The scope of our generalization is unlimited and hence larger than the scope of evidence.

- **Low degree of Probability :** As the generalization of Simple Enumeration are based on uniform experience of some cases, we cannot be sure of the unobserved cases/instances possessing the same characteristics as the observed ones. Generalization such as - 'All crows are black' is accepted as true on the basis of observation, ie. direct evidence. But we cannot rule out the possibility of a contrardictory instance. Therefore it is said to be probable.

- **Value of Induction by Simple Enumeration :** The generalizations established by simple enumeration are not equally good that is to say some generalizations are good and some are bad. For example : "All crows are black" is a good one but "All swans are white" is a bad one. Mill and Bacon considers the process of Simple Enumeration as childish and unreliable. According to them the value of Simple Enumeration depends upon the number of instances observed. However they were wrong in saying because value of generalizations depends upon some more conditions. They are as follows:

1. **Wider Experience :** The generalizations of Simple Enumeration are based on wider experience. For example : All crows are black, is based on observation. When a large numbers of instances are observed, it is possible to come across contradictory instance if any.

   Example : we do not come across any non - black crows
   ∴ We conclude 'All crows are black.'

2. **Variety of experience :** Instead of observing maximum number of crows from one part of the world, if we observe some crows from different parts of the world then the generalization becomes more probable or reliable because we all are aware that sometimes the colour of the animals depends upon the climate or other conditions of that region.

   E.g. Some bears are black.

   ∴ All bears are black.

   Here this argument is bad because in polar region we find white bears due to climatic condition.

3. **Resemblances :** Value of simple enumeration is also affected by the nature of resemblances. For example - crows apart from being black, resemble each other in other physical characteristics also like pointed beak, clawed feet, etc. which are equally important characteristics of a crow.

Analogy : Analogy is a type of inductive reasoning. Analogy is a common man's inference in which the conclusion is drawn on the basis of observed resemblances (similarities). In analogy we proceed form particular to particular instance.

It may be defined as an **argument from known resemblance to further resemblance, that is to say, if two (or more) things resemble each other in certain characteristics and if one of them have further / additional characteristics, the other is also likely to have that characteristics.**

The form of analogical argument is as follows
A - is observed to have the properties P1, P2, P3, ...Pn
B - is observed to have the properties P1, P2, P3, ...Pn
A possess additional property ‘q’
∴ B also has the property ‘q’.

Example :

On the basis of the observed similarities between Earth and Mars, Lowell put forward an analogical argument.

Both Earth and Mars are planets.

They revolve round the Sun.

Both have water, moderate temperature and are surrounded by an atmosphere.

There is life on Earth

Therefore there is life on Mars.

The logical basis of the analogical argument is that the characteristics found together are likely to be connected with one another and therefore from the presence of one characterstic we infer the presence of another.

Value of Analogy : Some analogical arguments are good whereas some are bad. The soundness of analogical argument depends upon the following factors :

- **Relevant and important resemblances :** When the resemblance is in important and relevant characteristics, the analogial argument is good. For example : Lowell’s analogy about Life on Mars is good example because they both resemble each other in important characteristics and are also relevant to the characteristic infered i.e. existence of life, as we all know water, temperature and atmosphere is necessary for existence of life.

- **Important differences :** If the differences are in important aspects, then the analogical argument is bad.

- **For example :**
  Man and monkey, both have two legs, two eyes, two hands, one nose, two ears.
  Man can read and write.
  Therefore, monkeys can also read and write.

  In this example, there are many similarities between both man and monkey but the difference is very important, i.e. man is rational whereas monkey is not as rational as man and therefore it is a bad analogical argument.
  It is important to note that the conclusion established by analogical reasoning is always probable and never certain.

The Nature of Conclusion : on the basis of the resemblances the conclusion of analogical argument should not assert more than what is justified by the evidence. The example of Earth and Mars justify the inference that there is life on Mars. But if one claims that there are human beings on Mars then the argument becomes a bad one, as too much is claimed than the evidence in the premises.

> *** Activity :** Recognize whether the given analogical arguments are good or bad and give justification for the same -

Examples :

1. Daniyal and Anita reside in the same building, they go to the same college and are in same class. They are of the same height and weight. Daniyal is smart.

∴ Anita is also smart.

2. Last time I purchased a pair of jeans from the store, it lasted for 2 years. Today also I purchased a pair of jeans from the same store and they are manufactured by the same company. The material of these jeans is also similar to the earlier one therefore this pair of jeans will also last for 2 years.

Scientific Induction

The task of science is to understand and explain facts. Scientific induction maybe defined as, **"the process of establishing generalization on the basis of direct and indirect evidence."**

According to **Mill** and **Bacon**, "Scientific induction is the process of establishing generalization which expresses a causal relationship." This process involves the following stages -

1. Some instances are observed and it is found that they possess certain common properties.
2. A generalization is made that all the instances, of that kind have the same property.
3. The observed instance is analyzed to discover if there is a causal relationship.
4. Experimental method is used to verify and establish the suggested causal relationship.

We cannot accept Mill and Bacon's views about scientific induction. There are two reasons -

1. All scientific generalizations do not express causal relation. For eg. the generalization all bats are warm blooded is not causal, because the property of being warm blooded is not an effect of being a bat.
2. The experimental method can provide only direct evidence, but scientific induction is supported by indirect evidence too. For eg - All observed metals expand when heated. Here observation of metals is direct evidence, but scientifc generalization does not stand in isolation, it is supported by other generalizations or well established laws that is 'All gases expands on heating'. Such support by other generalization / laws forms indirect evidence for scientific generalization.

Simple Enumeration and Scientific Induction

Both induction per simple enumeration and scientific induction are process of inductive reasoning and they both establish generalization. The logical form of Simple enumeration and Scientifc induction is same ie. they both infer from some to all, observed to unobserved. But they differ in certain important characteristics. The generalizations by simple enumeration are based only on direct evidences whereas the generalizations of scientific induction are based on direct and indirect evidence. In simple enumeration no attempt is made to analyse the observed cases whereas in scientifc induction the observed instances are analysed. The generalizations by simple enumeration possess low degree of probability whereas the genralizations by scientific induction possess high degree of probability.

Hypothetico - Deductive Method (Scientific method) :

Scientific induction has limited application. It can be used for establishing only scientific generalization. It is not suitable for establishing theories nor can it be used for establishing conclusions about a particular case. So to overcome this problem we need a method which can establish all kinds of propositions. The hypothetico-deductive method fulfills these conditions. It is the scientific method.

This method uses both deductive and inductive reasoning. **Hypothetico-deductive method consists of formulating a hypothesis, deducing consequences from it and verifying those consequences by appeal to facts.** This method involves five steps. These are as follows.

1. **Observation and feeling of a problem :** The aim of science is to understand and explain facts. When the scientist comes across an unfamiliar situation and when a familiar solution cannot explain the observed facts then scientific investigation begins. For eg - In Kon Tiki expedition, sociologists observed that the ancient customs of people living on south sea islands and the people of South America are similar. The problem felt was - why there is a similarity in customs and tradition of people who live far away from each other?

2. **Formation of an initial hypothesis :** When the observed facts cannot be understood then the scientist puts forth a temporary solution to explain the observed facts. This tentative (temporary) solution is called hypothesis. After the problem was felt some sociologists suggested a hypothesis that - In ancient days people from South America must have come to south sea island and must have settled down on the island and therefore the customs are similar.

3. **Collection of additional facts :** After forming the initial hypothesis the scientist collects additional facts relevant to the Hypothesis. In kon tiki expedition, additional data regarding various routes and means of travelling the distance between South Amercia and south sea island were collected.

4. **Deductive development of the hypothesis :** This stage is not required in some cases of scientific investigation where hypothesis are directly verified i.e. either by observation or experiment and the hypothesis which cannot be verified directly the scientist make use of deductive reasoning. In this the scientists construct a deductive argument where they supposes the hypothesis to be true and using it as a premise, consequences are deduced from it. eg - As sociologist's hypothesis is not possible to verify directly, so to verify indirectly' consequences were deduced ie. if people of South America travelled to south sea island then they must have travelled only through sea route and that too in a primitive kind of boat because in ancient days only such type of primitive boats were available.

5. **Verification of hypothesis :** Indirect verification consist of finding out whether the deduced consequences take place. If the predicted consequence take place then the hypothesis is accepted and if not, then it is rejected or modificed. eg - in Kon Tiki expedition sociologists made a small primitive kind of boat and actually travelled from South America to south sea island and they could travel this long distance. So they concluded that if we could travel this long distance today it is quite possible that in ancient days also people must have travelled and this explains similarity of customs.

PUZZLES

1. Dwayne Johnson was running away with the loot from a heist (robbery) in his car along with Vin Diesel. One tyre was punctured and he dropped down to replace it. While changing the wheel he dropped the four nuts that were holding the wheel and they fell into a drain. Vin Diesel gave him an idea due to which they were able to drive till their rendezvous point (destination). What was the idea?

2. A sweet girl purchased a book from a book keeper and gave him Rs 100. The cost of the book is Rs. 30 but the shopkeeper had got no change so he gets the change from the next shop and returns the girl her Rs. 70. After sometime the next shopkeeper comes with Rs 100 note and told the bookkeeper that the note is fraud. So he takes the money back. How much loss did the shopkeeper face?

3. Famous Elevator puzzle : A man who lives on the tenth floor takes the elevator down to the first floor every morning and goes to work. In the evening when he comes back, on a rainy day or if there are people in the elevator he goes to the 10th floor directly. Otherwise he goes to the 7th floor and walks up three flights of stairs to his apartment. Can you explain why?

Summary :

Inferences are classified into Deductive and Inductive.

Non deductive inferences are classified as :

- Simple enumeration.
- Analogy.
- Scientific induction.
- Hypothetic Deductive method.

Exercises

Q. 1. Fill in the blanks with suitable words from those given in the brackets :

1. In .... inference the conclusion asserts something more than what is given in premises. *(Deductive / Inductive)*

2. .... is called as a common man's method of arriving at a generalization.

   (Analogy / Simple enumeration)

3. .... is known as an argument from known resemblances to further resemblances. *(Analogy / Simple Enumeration)*

4. .... possess highest degree of probability. *(Scientific Induction / Simple enumeration)*

5. The process of arriving at generalization in science is known as .... *(Simple Enumeration/ Scientific Induction)*

6. Generalizations in science are supported by .... evidence. *(Direct / Both direct and indirect)*

7. .... is an inference from particular to particular. *(Analogy / Simple Enumeration)*

8. .... method uses both deductive and inductive reasoning. *(Simple Enumeration / Hypothetico-deductive method)*

9. .... verification consists of finding out whether the deduced consequences have taken place. *(Indirect / Direct)*

10. .... is a tentative solution. *(Hypothesis / Verification)*

Q. 2. State whether the following statements are True or False :

1. Induction is concerned with the form and not the content of an argument.
2. The generalization established in Simple Enumeration is based on uniform experience.
3. In Simple Enumeration we establish a proposition of restricted generality.
4. An Analogy is a deductive inference.
5. The generalizations established by Scientific Induction are certain.
6. Analogy involves an inductive leap.
7. The important difference between two objects does not affect the value of analogy.
8. Analogy is a deductive inference.
9. In Simple enumeration attempt is made to analyse the observed cases.
10. Hypothetico-deductive method consists of formulating a hypothesis, deducing consequences from it and verifying those consequences by appeal to facts.

Q. 3. Match the columns :

	(A)		(B)
1.	Scientific Induction	a.	Formal validity
2.	Simple Enumeration	b.	Temporary (tentative) solution
3.	Analogy	c.	High degree of probability
4.	Deductive Argument	d.	Based on resemblances
5.	Inductive Argument	e.	Material Validity
6.	Hypothesis	f.	Uniform experience

Q. 4. Give logical terms for the following

1. The inference in which we proceed from particular to particular instance.
2. A jump from known to unknown cases.
3. The method in which the generalization is established on the basis of uniform or uncontradictory experience.
4. The method in which the observed instances are analysed.
5. The scientific method in which both deduction and induction is involved.
6. The method in which the conclusion is based on the resemblances between two instances in certain qualities.

Q. 5. Give reason for the following :

1. There is a need for induction.
2. The method of Simple Enumeration has low degree of probability.
3. Conclusion of scientific induction has high degree of probability.

Q. 6. Explain the following :

1. Induction by Simple Enumeration.
2. Difference between Simple Enumeration and Scientific Induction.
3. The nature of analogy.
4. Value of sound analogy.

Q. 7. Answer the following questions :

1. Explain the characteristics Simple Enumeration.
2. What is Hypothetico-deductive method? and explain it's stages.
3. Explain with illustration vlaue of Simple Enumeration.
4. Explain the method of Scientific Induction.

Chapter 6 Fallacies

Logical fallacy is a flaw in reasoning. Logical fallacies are like tricks and illusion of thoughts.

Let us understand.

Argument - I

Aunt says : "Tony, do not smoke because parents don't like their children smoking. Don't You care for their emotions?"

Argument - II

Uncle says **:** "Tony, do not smoke because cigarette contains tobacco which is injurious to health.

Whose argument, do you think is correct? why?

....

6.1 MEANING AND DEFINITION OF FALLACY :

We all strive to reason correctly, but we do make errors in our reasoning or reason incorrectly.

We reason incorrectly when the premises of an argument fail to support its conclusion, then the arguments of this type are called as fallacious arguments. So in general any error in reasoning is called fallacy.

In I. M. Copi's words **'Fallacious arguments are those which appear to be correct but that are proved upon examination, not to be so.'** Notion of fallacy is therefore psychological in logic.

The term 'Fallacy', can be used for both deductive invalidity as well as inductive weakness.

6.2 PURPOSE OF STUDYING FALLACIES :

Study of fallacy helps us :

1. To realize that there are errors in our argument, to spot poor reasoning and most importantly to understand them.
2. Awareness of the fallacy brings closer to the truth or the situation.
3. Recognizing fallacies in arguments will help, to avoid committing errors in our own argument.
4. Lastly one can detect fallacies in others argument. So that, the person is not mislead by others.

6.3 CLASSIFICATION OF FALLACIES :

Complete classification of fallacies is not possible, as innumerable fallacies can be committed.

In Logic Fallacies are classified into two broad groups :

(1) Formal (2) Non-Formal

(1) Formal fallacy :

The formal fallacy is related to the structure of an argument. Logic deals with various forms of arguments. Validity of a deductive argument depends on the form/structure of an argument, it is governed by certain rules. Formal fallacy is committed, when a rule of logic is violated.

Activity : 1

e.g. (1) $p \supset q$ e.g. (2) $G \supset K$

p K

$\therefore q$ $\therefore G$

In the above examples which argument do you think is fallacious? why?

....

(2) Non-formal fallacy :

Non-formal fallacy is related to the content of an argument. Validity of Inductive argument depends on the content or subject matter of an argument and it is called material validity. Non-formal fallacies are committed due to misleading use of language.

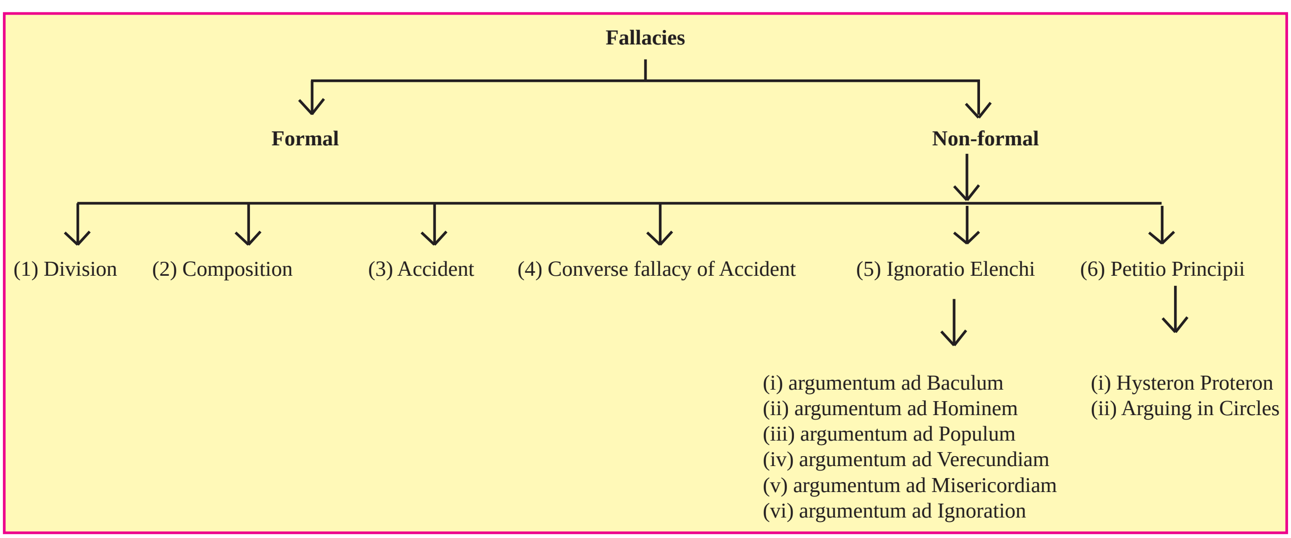
i.e. (1) Irrelevant conclusion.
(2) Ambiguous use of words.
(3) Wrong use of collective and distributive terms.
(4) Wrong use of rules in exceptions.

6.4 CLASSIFICATION OF NON-FORMAL FALLACIES :

Non-formal fallacies may be further classified as :

(1) Fallacy of Division
(2) Fallacy of Composition
(3) Fallacy of Accident
(4) Converse fallacy of Accident
(5) Fallacy of Ignoratio Elenchi [Irrelevant Conclusion]
(6) Fallacy of Petitio Principii [Begging the Question]

Chart of Fallacies :



(1) Fallacy of Division :

In fallacy of Division one wrongly proceeds from collective use of the term to distributive use.

When all the members of a class taken together, possess certain quality then the term is to be used collectively.

E.g. The weight of mangoes in the basket is 5 kgs. Here all mangoes in the basket taken together are 5 kgs. Here the term 'weight' is used collectively.

Distributive term means, each member of a class individually has certain quality.

E.g. When we say all mangoes in this basket are sweet. Here we mean each Mango is individually sweet. Thus the term sweet is used distributively.

The term fallacy of Division arises in two ways :

(1) From class to member

For instance, It would be fallacious to argue that because the 'College cricket team is good. Hiten being the member of the college cricket team, Hiten is good player.

(2) From whole to part.

For instance, It would be fallacious to argue that because an object i.e. 'Machine' as a whole is heavy. Therefore each and every part of the machine is heavy.

Definition : The fallacy of Division is committed, when it is wrongly argued that what is true of a class is also true of its member separately, or what is true of the whole is also true of its part singly.

Examples :

(i) A bag full of rupee coins, is heavy. Therefore each and every rupee coin in it is heavy.

In this example it is wrongly argued that what is true of 'all rupee coins collectively in a bag', i.e. it is heavy, is also said to be true of each rupee coin, in that bag.

(ii) Water is a liquid. Therefore its constituents Hydrogen and Oxygen are also liquids.

In this example it is wrongly argued that what is true of 'water' as a whole i.e. it a liquid, is also said to be true of its parts i.e. constituents Hydrogen and Oxygen separately.

Activity : 2

Anita lives in a large building. So her apartment must be large.

Why do you think that the fallacy of Division is committed in the above example? Explain.

....

....

(2) Fallacy of Composition :

In the fallacy of Composition, one wrongly proceeds from distributive use of a term to its collective use. The fallacy of Composition is opposite to the fallacy of Division. The term fallacy of Composition, also arises in two ways :

(1) From member to class.

For instance : It would be fallacious to argue that because a child from the class is physically weak, therefore the class (group) of children is also physically weak.

(2) From part to whole.

For instance : It would be fallacious to argue that because each brick as the part of the building, is light in weight therefore the building as a whole is also light in weight.

Definition : Fallacy of Composition is committed, when it is wrongly argued that what is true of each member separately, is also true of the class or what is true of each part singly is also true of the whole.

Examples :

(i) Orange juice is tasty, Ice-cream is tasty and fish curry is tasty. Therefore the mixture of all the three ingredients is bound to be tasty.

In this example it is wrongly argued that what is true of each ingredient separately i.e. it is tasty, is also said to be true of the mixture, collectively prepared with it.

(ii) Each chapter of this book is small. Therefore this book is small.

In this example it is wrongly argued that what is true of each chapter as the part of the book i.e. singly it is small, is also said to be true of the whole book.

Activity : 3

Seeta, Geeta and Neeta of class XI A are intelligent. Therefore class XI A is an intelligent class.

Why do you think that in the above example, fallacy of Composition is committed? Explain.

....

....

(3) Fallacy of Accident :

This fallacy arises due to wrong use of rules in exceptions.

There are certain moral, legal, educational or social rules or principles. Such rules are in normal circumstances desirable. But from this it does not logically follow that they should be applied even in special cases. In other words practically every rule has exceptions. It is not applicable in special, accidental or exceptional circumstance.

Definition : When it is argued that what is true as a general rule, is also true in a special case, the fallacy of Accident is committed.

Examples

(i) Regular walk is good for keeping oneself physically fit.

Therefore, a patient with fractured leg must also walk regularly.'

In this example the general rule i.e. Regular walk is good…., is applied to a special case of a patient with fractured leg. Hence the fallacy of accident is committed.

(ii) One should always speak the truth. Therefore the doctor is wrong, when he tells the terminally ill patient that there is improvement in his health and he will be fine very soon.

In this example, the general rule i.e. 'One should speak the truth'cannot be applied to a special case of a terminally ill patient. Hence the fallacy of accident is committed.

Activity : 4

It is wrong to shed blood. Therefore a surgeon should not perform an operation on a patient.

Why do you think that in the above example, the fallacy of Accident is committed? Explain.

....

....

(4) Converse fallacy of Accident :

This fallacy arises due to wrong use of rules in exceptions.

This fallacy is the converse of the fallacy of Accident. According to Cohen and Nagal there are certain truths which are "accidental truths." It is irrelevant to arrive at general principles out of accidental truths, In other words, what is true in accidental or exceptional case need not be true in general. In this fallacy an attempt is made to arrive at a general rule on the basis of a special or an exceptional case.

Definition : when it is argued that what is true in a special or exceptional case, is true as a general rule, we commit the Converse fallacy of Accident.

Examples.

(i) An ambulance is allowed to overtake other vehicles and break traffic rules. Therefore every vehicle must be allowed to break traffic rules.

In this example, it is argued that what is true in a special case of ambulance i.e. it is allowed to break traffic rules is accepted as a general rule for all vehicles.

(ii) A visually challenged student is given a writer for the exam. So every student must be given writers for exams.

In this example it is argued that what is true in a special case of a visually challenged student i.e the student is given a writer for the exam is accepted as a general rule for all students.

Activity : 5

Mr. X died while performing an operation on him. So Surgeons must not be allowed to perform operations on patients

Why do you think that in the above example the Converse fallacy of Accident is committed? Explain.

....

....

(5) Fallacy of Ignoratio Elenchi [Irrelevant Conclusion]

Ignoratio Elenchi is the latin expression. It is called the fallacy of "irrelevant conclusion".

In this fallacy, the conclusion is irrelevant, i.e. the premises are besides the point. So they do not yield the conclusion. The argument is put across in such a way that the listeners may be misguided to accept it as good. Ignoratio Elenchi is a group of fallacies. Let us study each fallacy in detail.

1. Argumentum ad Baculum (Appeal to threat, fear and force)

In this fallacy, there is an appeal to force or fear in order to get an argument accepted by the opponent. The appeal need not be always to physical force, but it may be in a non-physical manner, in a more minute way, in the form of mental torture, i.e. social boycott, or even threat of war. Anything which arouses fear in an opponent, that forces the opponent to accept it. In logic our conclusion is correctly drawn only when we give good reasons for it.

Definition : The fallacy of argumentum ad Baculum is committed, when the person does not have rational argument and instead he appeals to threat, fear and force to establish his conclusion.

This fallacy is based on the principle of **'Might is right'**.

Examples

(i) Teacher to the student "If you do not attend the lectures, I will fail you."

In this example the teacher threatens the student that she will fall him. This creates fear in the mind of the student, so he is forced to attend the lectures.

(ii) An industrialist to his employees : "If you join the union, I may seriously think of terminating you from the job,"

In this example an Industrialist threatens his employee that he will terminate the employee from the job, which creates fear in the mind of the employee and he is forced not to join the union.

Activity : 6

The soldier to his enemy at war: "Surrender or Die".

Why do you think that in the above example the fallacy of Argumentum ad Baculum is committed? Explain.

....

....

2. Argumentum ad Hominem [Appeal direted against the man]

It is a very ancient but very prevalent fallacy even in recent times.

Definition : The fallacy of argumentum ad Hominem is committed when a person, instead of giving correct reasons to prove ones own argument, makes an attempt to refute an opponent's argument by a personal attack on the opponent's character, conduct, reputation [beliefs or opinions], background, or past views which are irrelevant to the situation.

The term 'Argumentum ad Hominem' literally means 'Against the man'. Modern logicians called it, the fallacy of **'tu quoque'** which means **'you also'**.

Mostly this fallacy is committed in a courtroom, in the field of politics and debates..

Examples

(i) "How can you talk in favour of co-education, when you send your daughter to girls college."

In this example the person attacks the opponent's conduct i.e. sending his own daughter to girls college, instead of proving his argument with proper reasons.

(ii) "What right do you have, to tell me, to wear a helmet while riding a bicycle, when I have never seen you wearing it?"

In this example the person attacks the opponent's conduct i.e. not wearing a helmet while riding a bicycle, instead of proving his own argument with proper reasons.

Activity : 7

"How can you accuse me for copying in the exam, I had seen you copying in the exam last time."

Why do you think that fallacy of Argumentum ad Hominem is committed in the above example? Explain.

....

....

3. Argumentum ad Populum. [Emotional appeal to people]

Irving Copi calls it "an emotional appeal to people". In this type of fallacy appeal is made to people's emotions rather than to reason, in order to establish one's own point of view.

Definition : When the premises of an argument make an appeal to people's emotions, and feelings in order to support the truth of some unrelated conclusion, the fallacy of argumentum ad Populum is committed.

Propagandists use this type of arguments as easiest way of arousing people's emotions. For this purpose they may sometimes use emotional language which is irrelevant to the content or information. e.g. Political parties using emotional language to win the votes.

Examples

(i) A particular model of mobile is the best in the market Don't you know that it has the highest sale in the market?

In this example, there is an emotional appeal to people for a particular model of mobile.

(ii) "How can you criticize Dowry system? Are you wiser than your ancestors?"

In this example there is an emotional appeal to people, to follow Dowry system.

Activity : 8

"Married Girl must wear sari. Don't you know that the great grand-mothers always did that for years?"

Why do you think that in the above example the fallacy of Argumentum ad Populum is committed? Explain.

....

....

4. Argumentum ad Verecundiam. [Appeal to improper authority]

We cannot always prove everything by our-self. So we have to accept the views of an authority. But very often, the authority we quote is not a proper one.

Definition : The fallacy of argumentum ad Verecundiam is committed when an appeal is made to improper authority.

The person sometimes does not have special knowledge in the area of discussion, yet to prove one's own point of view, an improper authority is quoted.

Advertisers takes advantages of the popularity of some famous personalities for the sale of their products. When common people are made to believe and accept that a particular product is good just because famous people recommended it, this fallacy is committed.

Examples

(i) A famous film star claims, a particular hair oil is the best. So it must be good.

In this example, the argument appeals to the authority of an actor. But actor is an improper authority for deciding whether the hair oil is good.

(ii) I am sure that, this cold drink is a very good drink, as I heard the famous cricketer talk about it, in one advertisement.

In this example, the argument appeals to the authority of a cricketer. But cricketer is an improper authority for deciding whether the cold drink is good.

Activity : 9

How can you doubt it ? My friend said, the film is good.

Why do you think that in the above example the fallacy of Argumentum ad Verecundiam is committed? Explain.

....

....

5. Argumentum ad Misericordiam. [Appeal to pity]

In this fallacy there is an appeal to pity or feeling of sympathy for getting a conclusion accepted. However an appeal to pity is not always logically relevant to the truth of a conclusion.

Definition : The fallacy of Argumentum ad Misericordiam is committed when someone tries to win support for an argument by making an appeal to feeling of pity or sympathy.

It is very common in court room. When the defence attorney is unable to offer good reasons for his client's defence, he may appeal to pity as a last attempt to save the client from being punished

Examples

(i) "Gentlemen of Jury, I earnestly make an appeal to you to sympathize with my client - who is a pretty young widow, with tear-stained face, mourning and holding a new born baby in her arms."

In this example the lawyer tries to win support for his client by an appeal to the feeling of pity. So that the Jury will forgive his client.

(ii) "Sir, I request you to pardon me. No doubt I am guilty of copying in the examination, but you know that my father is no more and my mother has been suffering from cancer since last two years. I being the eldest in the family, had to look after my sick mother and younger siblings. So I could not prepare for the examination."

In this example the student tries to gain support, by an appeal to the feeling of pity for himself, so that the teacher will forgive him.

Activity : 10

"Please do not dismiss me from job, I really need it. My Father is now bed-ridden. I am the only son and have to look after my old parents."

Why do you think, the fallacy of Argumentum ad Misericordiam is committed in the above example? Explain.

....

....

6. Argumentum ad Ignorantiam [Appeal to ignorance]

Definition : The fallacy of argumentum ad Ignorantiam occurs, when lack of knowledge or ignorance of the opponent is taken as evidence to prove one's own point of view.

In other words it is an error that is committed, when it is argued that one's proposition is true, simply on the basis that it has not been proved as false by the opponent or the opponent's proposition is false simply because the opponent has no evidence to prove it as true.

Here ignorance of how to prove or disprove a proposition, clearly does not establish the truth or falsity of the proposition. This kind of argument is not a fallacious in the court of Law because as per the guiding principle, court assumes that person is innocent till he is proved to be guilty.

Examples

(i) Mr. Peter said, he is courageous because nobody ever told him, he is not.

In this example there is an appeal to ignorance. i.e. Just because the opponents have no evidence to prove that 'Peter is not courageous', therefore Peter's statement that 'he is courageous' is considered as true.

(ii) Nobody has so far proved that ghost exists. Therefore ghosts do not exist.

In this example there is an appeal to ignorance, i.e. Just because there is no evidence to prove that 'ghost exists, therefore ghosts do not exist is considered to be true.

Activity : 11

Nobody has so far proved that the soul is mortal. Therefore the soul is immortal.

Why do you think that in the above fallacy of Argumentum ad Ignorantiam is committed? Explain.

....

....

(6) Fallacy of Petitio Principii [Begging the Question]

Petitio Principii is a fallacy of proof rather than inference. Here it should be noted that the premise is not logically irrelevant to the truth of the conclusion but the premise is logically irrelevant to the purpose of proving or establishing the conclusion.

Petitio principii is popularly known as 'Begging the Question'. The expression 'begging the question' makes it clear that which is to be proved, is taken for granted.

E.g. To give charity to beggars is right because it is the duty to be charitable. Here premise contains the conclusion. So the fallacy of Petitio principii is committed.

It takes two sub-forms :

1. Hysteron Proteron :

In Hysteron Proteron, there is direct assumption. This fallacy is committed in a single step of inference by use of synonym. That means the reason given [i.e. the premise] merely repeats the statement to be proved [i.e. conclusion] but in different words, having the same meaning.

Examples

(i) This cloth is transparent
Because we can see through it.

In this example, the premise i.e. we can see through the cloth repeats the conclusion i.e. the cloth is transparent, in different words having the same meaning.

(ii) The wind is invisible
Because we can never see it.

In this example, the premise i.e. we can never see wind repeats the conclusion i.e. the wind is invisible, in different words having the same meaning.

Activity : 12

Mr. Raju is insane, for his behaviour is that of a mad man.

Why do you think that, in the above example the fallacy of Hysteron Proteron is committed? Explain.

....

....

2. Arguing in Circles.

In fallacy of Arguing in Circles, or Vicious Circle, the premise that is assumed is not the conclusion itself. But it is something whose proof's depends upon the conclusion.

Here the subject of the premise becomes the predicate of the conclusion and vice versa.

The logical form of this fallacy is : P is true, because Q is true.

And Q is true, because P is true.

Examples

(i) Monica is famous, therefore she is in film industry.

Monica is in film industry, therefore she is famous.

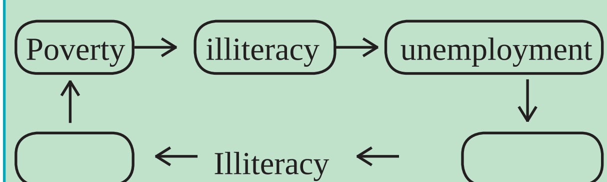
In this example, the premise i.e. Monica is famous, therefore she is in film industry repeats itself as a conclusion i.e. Monica is in film industry, therefore she is famous, but in a round about manner.

(ii) Healthy mind implies healthy body, and Healthy body implies healthy mind.

In this example, the premise i.e. Healthy mind implies healthy body repeats itself as the conclusion i.e. Healthy body implies healthy mind, But in a round about manner.

Activity : 13

Complete the vicious Circle.



Summary

- Fallacy means an error in an argument.
- Fallacies are classified into two types formal and non-formal.
- Formal fallacy is committed, when a rule of Logic is violated
- Non-formal fallacy in committed due to misleading use of language.
- I. M. copi has classified non-formal fallacies as follows :
  - (1) Fallacy of Division
  - (2) Fallacy of Composition
  - (3) Fallacy of Accident
  - (4) Converse Fallacy of Accident
  - (5) Fallacy of Ignoratio Elenchi
    - (i) Argumentum ad Baculum
    - (ii) Argumentum ad Hominem

(iii) Argumentum ad Populum

(iv) Argumentum ad Verecundiam

(v) Argumentum ad Misericordiam

(vi) Argumentum ad Ignoratiam

(6) Fallacy of Potitio principii

(i) Hysteron Proteron

(ii) Arguing in Circles

Activity : 14

Give few examples of the experiences you had, where you came across some fallacies committed in the arguments.

Exercises

Q. 1. Fill in the blanks with suitable words from those given in the brackets :

(1) Fallacy of .... is committed, when it is argued that what is true as a general rule, is also true in a special case.

[A] Accident

[B] Converse of accident

(2) .... fallacy is committed, when the rule of Logic is violated.

[A] Non-formal *[B] Formal*

(3) Fallacy of argumentum ad ...., is committed when we appeal to pity.

[A] Verecundiam *[B] Misericordiam*

(4) Fallacy of argumentum ad ...., is based on the principle of 'Might is Right'.

[A] Baculum *[B] Populum*

(5) Fallacy of .... is committed when it is wrongly argued that, what is true of the whole class is also true of its member.

[A] Division *[B] Composition*

(6) Fallacy of .... is committed when the premise repeats itself as the conclusion, in a round about manner.

[A] Hysteron Proteron

[B] Arguing in Circles

(7) When there is an appeal to ...., the fallacy of argumentum ad Verecundiam is committed.

[A] Improper authority

[B] Emotional feeling of people

(8) When an argument wrongly proceeds from .... of term, the fallacy of Composition is committed.

[A] Collective use of term to distributive use

[B] Distributive use of term to collective use

(9) The fallacy of argumentum ad .... is committed when it is argued that a proposition is true simply on the basis that it has not been proved false or that it is false because it has not been proved to be true.

[A] lgnoratiam *[B] Hominem*

(10) In the fallacy of .... there is direct assumption.

[A] Hysteron Proteron

[B] Accident

Q. 2. State whether the following statements are true or false:

(1) Argumentum ad Hominem occurs when one attempts to attack on personal drawbacks and short comings of man.

(2) In the fallacy of Petitio principii, what is to be proved is taken for granted.

(3) Converse fallacy of Accident is committed when it is argued that what is true of a member of a group is also the of the whole group.

(4) The modern name for the fallacy of argumentum ad Populum is 'tu quoque'.

(5) Fallacy of argumentum ad Misericordiam is committed when we appeal to threat.

(6) Formal fallacy is committed, when the conclusion is not relevant to the premise.

(7) Fallacy means an error in an argument.

(8) In the fallacy of Petitio principii the premise is logically irrelevant to the purpose of proving or establishing the conclusion.

(9) Fallacy of Division is generally committed in the court-room, as a last attempt to save the client from being punished.

(10) Argumentum ad Ignoratiam is not fallacious in the court of Law.

Q. 3. Match the columns:

	(A)		(B)
(1)	Ignoratio Elenchi	(a)	Emotional appeal to People
(2)	Petitio Principi	(b)	Appeal directed against the man
(3)	Argument Ad Hominem	(c)	Irrelevant Conclusion
(4)	Argument Ad Populum	(d)	Begging the Question

Q. 4. Give logical terms for the following :

(1) Any error in reasoning.

(2) Error due to misleading use of language.

(3) Error done due to violation of any rule of logic.

(4) Error done in the argument, where one wrongly proceeds from part to whole.

(5) Error done in an argument, where one arrives at a general principles on the basis of accidental cases.

(6) Latin expression for the fallacy of 'irrelevant conclusion'.

(7) The error done in an argument, where one threatens his opponent and forces him to accept his statement as true.

(8) The fallacy of an argument against the man.

(9) Fallacy that is used as a last resort in the court-room to save the client from being punished by mercy petition.

(10) The error committed in an argument for the sale of products by the celebrities.

Q. 5. Explain the following :

(1) Fallacy of Division

(2) Fallacy of Composition

(3) Fallacy of Accident

(4) Converse fallacy of Accident

(5) Fallacy of argumentum ad Baculum

(6) Argumentum ad Hominem

(7) Argumentum all Verecundiam

(8) Argumentum ad Ignoratiam

(9) Argumentum ad populum

(10) Petitio Principii

Q. 6. Recognize with reasons, the fallacies committed in each of the following arguments :

(1) "If you do not vote for my candidate, then you will find it difficult to stay in this locality

(2) The ball is blue. Therefore the atom that make it up are also blue.

(3) How can Ravi be truthful? Because his own brother was caught for lying.

(4) We should never treat any human being as a means. Therefore we should not hire a coolie for lifting our heavy luggage.

(5) The novel is interesting because many people read it and the novel is read by many people because it is interesting.

(6) It is meaningless to argue in favour of democracy, since even a famous cricketer was against it.

(7) Employee to his Boss says "Sir I appeal you, not to dismiss me from my job. I have to support my old parents and young children. If I lose my job, my family will have to starve to death. So please have pity on me."

(8) Mr. X is humiliating because he is degrading.

(9) A student is allowed to appear for supplementary examination, as she was sick during terminal examination. So all the students must be allowed to appear for supplementary examination.

(10) These documents are authentic because they are factual.

(11) The Union has voted for strike. As a member of the Union, you too must have voted for strike.

(12) He cannot be successful home minister. Since he could not even manage his own family affairs.

(13) This Airline service is the best in the world. Don't you know that they have been serving people, since last one decade?

(14) We had this law for last forty years, but nobody talked against it. So this Law is correct.

(15) "Mam please assess my answer sheet again, there may be some error. I studied very hard for weeks and my career demands on getting a good grade. If you give me a failing grade I'm ruined! Please have pity on me."

(16) "To defend oneself from injury is perfectly justified. Therefore a patient is justified in kicking a surgeon who is about to perform an operation on him.

(17) A Girl to a friend says "If you do not come with me for the movie, I will not talk to you."

(18) Soul is eternal as it never dies.

(19) I am sure our party will win this election as the famous actor said so in his recent meeting.

(20) Soldiers are right in killing the enemy at war. Therefore, we should not object to soldier killing people.

(21) Each student of this class is attentive. Therefore this class on the whole is attentive.

(22) There is no evidence to prove that there is life after death. Therefore there is no life after death.

(23) 'Accident caused by youngster's driving', is commonly read in Newspapers. Hence no youngster should be allowed to drive.

(24) Sodium Chloride [table salt] may be safely eaten. Therefore its constituent elements, Sodium and Chloride can also be eaten safely.

(25) How can you believe the charge made against the CEO of the company, when the person making the charges himself is a culprit?

(26) "Ladies and gentlemen of the jury, look at this miserable man, in a wheelchair, unable to use his legs. Could such a man really be guilty of embezzlement?"

(27) "If you do not promote the sales, then you will be dismissed from the job."

(28) "Artists are moody. Hemant is an artist. Hence Hemant is moody."

Chapter 7 Application of Logic

The better you are at logic, the more likely you are to be the master of your own life than its victim.

DO YOU KNOW THAT ....

Skills of logic can never be out – dated.

Knowledge of principles of logic is the key to successful life.

Logic has applications in all fields of life.

Logical thinking is wider than scientific thinking.

Computer science is based on principles of logic.

Logic is essentially the study of reasoning or argumentation. We all use reason all the time to draw inferences that are useful to us. Study of logic grooms us to construct good arguments and to spot bad ones. This is a skill that is useful in every field as well as in everyday life. Let us learn application of logic in some important fields like – Law, Science, Computer science and everyday life.

7. 1. APPLICATION OF LOGIC IN EVERYDAY LIFE

To comprehend is essentially to draw conclusions from an already accepted logical system – Albert Einstein

Logic is useful in our everyday life in many ways. **In our daily affairs we have to make many decisions and decision making is not possible without logic.** Every day we come across many situations, problems, or challenges that may be trivial or serious. For instance, simple situations where a housewife has to choose a grocery shop to buy quality products, or choose a juicer of a certain company from various available brands in market, or an important and challenging situation before a youngster, of choosing a career or a life partner.

Good or valid reasoning is necessary to take correct decisions in such situations. Irrational decisions, influenced by advertisements, emotions, biased opinions etc. are not useful. For example a student, who has passed S. S. C with good marks, has to decide about his career before taking admission for science, commerce or arts. His decision may get influenced by many factors like – popular trend in society to become say an engineer, parents desire of their child becoming a doctor, pressure from friends, where all friends are taking admission for commerce, relatives saying no for taking arts, and his own wish to become a singer. In such a situation one needs to think logically, by analysing the situation, finding out various options available, deciding the priorities, understanding one's own interests, talents, abilities, and aptitude for a certain field. For this one can even seek vocational guidance. And finally one arrives at a right decision. **Logical thinking thus helps us to take right decisions at right time, which in turn can make us successful in all spheres of life. And success in life gives us confidence in our innate powers to think rationally.**

Logical thinking is analytical or inferential thinking. It has to be developed with proper guidance and training. Logic cultivates the power to understand abstract concepts. With maturity one improves and with practice one can strengthen these powers. This is the reason why all competitive exams like – C. A, Law, UPSC, MPSC etc. have one paper of reasoning to test students reasoning ability.

This does not, however, imply that without formal training in logic one cannot reason logically. Logical reasoning in fact is an inbuilt feature of human mind. Study of **logic only makes one better or well – equipped to reason correctly than the person who has not studied logic.**

Logic is useful in communication and conveying. One of the important purpose of language is to communicate our thoughts, ideas, opinions, and feelings with other people. Knowledge of logic can make our communication more precise and perfect, by enhancing our ability to express ideas clearly and concisely. To make people understand what we wish to convey. It is necessary that the subject matter is expressed in logical order, there are no inherent inconsistencies and the important points are highlighted with logical justification. This will help us not only to convey our ideas, thoughts or feelings precisely but also to convince people.

Knowledge of principles of logic enables us to evaluate and critically analyse others arguments. It also develops our ability to formulate argument rigorously. In our day to day life many arguments by people from various fields attract our attention like – a salesman persuading us to buy product of a certain company, an advertisement telling us how a particular product is good and should be preferred over other similar products, friends / parents / relatives advising us regarding an important decision in life, a politician convincing us for giving vote for him and his party. Knowledge of rules of logic and fallacies empowers us to evaluate such arguments and decide if they are good or fallacious. Logic also helps us to formulate correct arguments and avoid fallacies when we think, form opinions, reason, debate or argue with others. Thus logic helps us to refute others arguments and prove one's own argument easily.

Logic is also useful in discussions, when the aim is to understand the topic of discussion and arrive at some common agreement. knowledge of fallacies, definitions can help in gaining better insight into the topic and arrive at mutual agreement.

7.2 APPLICATION OF LOGIC IN LAW

Every legal analysis should begin at the point of reason, continue along a path of logic and arrive at a fundamentally fair result." (Sunrise Lumber V. Johnson, Appeal No. 165)

Knowledge of principle of logic empowers us to reason correctly by training us to differentiate between good and bad reasoning. This is very important and more clearly demonstrated in legal trials, than in any other field.

Evaluating and creating arguments is essential to the crafts of lawyering and judging. It is important for practitioners as well as students of the law to understand the basic principles of logic that are used regularly in legal reasoning and judicial decision making. This understanding includes. (1) Expertise in using inductive reasoning e.g. the methods of analogy and simple enumeration – by which inferences are drawn on the basis of past experience and empirical observation. "The Rule of Law" – that like cases be decided alike – is grounded logically in inductive reasoning. (2) Elementary understanding of deductive logic, especially of argument forms called 'syllogism', gives lawyers, judges, and students of the law a valuable tool for deciding whether an argument in a legal opinion is valid or fallacious.

To criticize, reserve, or overrule an administrative or judicial decision as "arbitrary," "capricious," "unsupported by law "or" contrary to precedent" is to say nothing more, but nothing less, than that the decision is deficient in logic and reason.

The role of logic is significant in all the three important aspects of legal system – making of laws, execution of laws and interpretation of laws.

The language used is very important while making legal laws. The laws should not be vague or ambiguous. They ought to be very clear and precise. The precision of details is also necessary in the drafting of contracts, wills, trusts and other legal documents. This is possible when the words used in laws are properly defined. Knowledge of principles of logic is important and necessary for making laws and legal documents.

Execution of laws is the essential aspect of legal system. The main function of judicial system is to resolve disputes. It is necessary that the judgement arrived at is definite and fair. The entire process of legal trial is based on application of principles of logic. Knowledge of different types of fallacies is very useful in legal trials. Knowledge of fallacies not only enables lawyers to detect errors in opponent's arguments, but it also helps them to argue correctly and justify one's own stand. Finally by applying principles of logic, argumentation of lawyers is evaluated, evidence before the court is weighed and a fair judgement is arrived.

For resolving disputes, sometimes, a legal system has to apply some law or a rule or a principle to a set of facts so that some judgement is possible. One lawyer for instance, to defend his client may claim that a specific rule applies to the facts whereas the opponent lawyer may claim that the rule does not apply. In such cases knowledge of logic is useful in correct interpretation of the law or rule.

7. 3 APPLICATION OF LOGIC IN SCIENCE

Science is defined as, **'A systematized body of factual knowledge collected by means of scientific method.'** Science is born out of man's inherent curiosity to explore and understand the world around him. Man's thirst for knowledge is to know 'true' nature of facts. Our understanding of facts, however, need not be always correct. So there is a need to have a test to distinguish between correct and incorrect explanations of facts. The explanations which are rational, logical and based on factual evidence are accepted as correct explanations in science.

The scientific method (Hypothetico deductive method) clearly illustrates how scientific thinking follows logical thinking. Every stage in scientific method has its basis in logic.

(1) The first important step in scientific method is – Formulation of hypothesis. Though the role of creative imagination is significant in suggesting a hypothesis, it is not a result of wild, but a logical guess. Deductive and inductive inferences like simple enumeration, analogy may suggest hypothesis to scientist.

(2) The suggested hypothesis should be a good hypothesis. To decide whether it is relevant, self – consistent, compatible with other laws, knowledge of rules and principles of logic is necessary.

(3) In order to verify the hypothesis, what are the relevant facts to be observed and data to be collected, whether the evidence collected is relevant and sufficient, what experiment to be conducted, all these decisions have basis in logical thinking.

(4) Most of the hypothesis are verified indirectly in science by deducing consequences form the hypothesis. Deductive reasoning is necessary for such deductive development of the hypothesis.

Our knowledge of logic further makes it evident that, indirect verification commits the fallacy of affirming the consequent. So the next step is to prove the hypothesis by showing that no other hypothesis can explain the facts except the proposed hypothesis. As one connot possibly know all alternate hypotheses, it is not possible to prove the hypothesis. Thus we logically come to the conclusion that scientific laws and theories cannot be conclusively proved and scientific knowledge is probable.

When any law or a theory is proved in science, only the evidence in its support is not enough, the proof / the argument should be valid. Knowledge of logic helps in deciding validity of argument.

(5) Scientific laws explain facts by introducing different types of orders into facts like – classificatory, causal, mathematical and order introduced by theories. All

these orders are arrangement of facts as per some plan which is based on logical thinking. Theories introduce order among laws which fall within its scope. This is the highest kind of order in science. It is also known as vertical organization in science. From theories laws can be deduced which in turn explain facts. This shows that science as a system is based on principles of logic.

Relation between scientific thinking and logical thinking is one sided. Logic helps science but science cannot be useful in logic. Logical thinking is wider than scientific thinking;rather scientific thinking is based on logical thinking. Technology which is application of scientific laws and theories is also based on logical consequences and predictions which are derived from scientific theories.

7. 4 APPLICATION OF LOGIC IN COMPUTER SCIENCE

Computer is the most significant invention of the 20th century. Computers have influenced our life to great extent. They are used at almost every workplace and home. Computers have becomes almost indispensable in modern man's life. Though computer appears to be superior to man, it cannot think and reason like man. It can only perform as per the instructions given to it. However what makes it a brilliant invention is the fact that it is considerably faster, accurate, and consistent than man. It can do multiple tasks at one time and unlike man it can function continuously for hours.

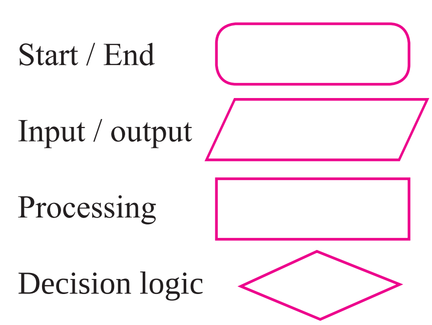
Computers can perform certain tasks and solve problems by carrying out instructions given to it. A sequence of instructions describing how to perform a certain task is called a program. Such a program is in a language which computer can understand. The language which computer understands is called. 'machine language'

Knowledge of principles of logic is necessary for making computer programs. Computer uses binary system for its operation. There are only two digits 0 and 1. One of the reasons for this is human logic tends to be binary – true or false, yes or no statements. Information which is in language is coded in binary digits to feed it to the computer. After processing the output given by computer is also in binary digits, which is displayed on the screen by converting it into language.

A computer thus receives stores, understand and manipulates information composed of only 0 and 1. The manipulation of binary information is done by logic circuits known as logic gates. The important logic operation which are frequently performed in the design of digital system are – AND, OR, NOT, NAND, (NOT – AND), NOR and EXCLUSIVE – OR. These logic gates are the basic building blocks of computer. A logic gate manipulates binary data in a logical way. The knowledge of logic gates is essential to understand the important digital circuits used in computers like – addition, subtraction, multiplication. The input output relationship of the binary variables for each gate can be represented in a tabular form by a truth table which is essentially same as truth tables used in logic.

To solve any problem, programmer provides a method to the computer. It is in form of a procedure which is a series of steps in a logical sequence. This is called an algorithm. Algorithm is expressed in form of flow chart, which is essentially a diagram that defines the procedure. A flow chart shows the order of operations and the relationship between the sections of the programs. Flow charts are independent of a particular computer or computer language.

There are some standard symbols which are usually used in drawing flow chart like –



The flow chart to calculate sum of two numbers, for instance, is as follows –

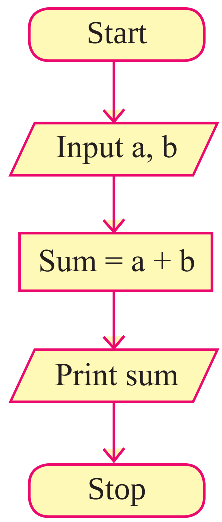
Step 1 : Input two numbers a & b

Step 2 : Calculate sum = a + b

Step 3 : Print sum

Step 4 : Stop

Flow chart –



Summary

- Logic trains us in valid reasoning. This ability to reason correctly is useful in every sphere of life.
- In everyday life, logic empowers us to take correct decisions, which in turn leads to success in life and develops confidence in rational thinking.
- Logic is useful in communication.
- Principles of logic enable us to critically evaluate others as well as one's own arguments.
- Role of logic is important in legal trial. Logic is useful in making of laws, execution of laws and interpretation of laws.
- Scientific, method follows logical thinking. Every stage in scientific thinking has basis in logic.
- Logical thinking is wider than scientific thinking.

Exercises

Q. 1. Fill in the blanks with suitable words in the brackets.

(1) Knowledge of …..……. can make our communication more precise and perfect. *(Psychology / Logic)*

(2) Formal training is …..……. to reason logically. *(Necessary / Not necessary)*

(3) Knowledge of principles of logic enables us to evaluate and critically analyse others …..……. *(Arguments/ Emotions)*

(4) Knowledge of …..……. develops our ability to formulate valid arguments. *(Fallacies / Law)*

(5) Hypothesis is a …..……. guess. *(Wild / Logical)*

(6) Logical thinking is …..……. than scientific thinking. *(Narrower / Wider)*

(7) A sequence of instruction describing how to perform a certain task is called a …..……. *(Program / Process)*

(8) Computer uses …..……. system for its operation. *(Monadic / B inary)*

Q. 2. State whether following statements are true or false.

(1) Logical thinking helps us to take right decision.

(2) Logic gives us confidence in our innate powers to think rationally.

(3) Logic is not an inbuilt feature of human mind.

(4) Logic is not useful in communication and conveying.

(5) Inductive inferences like simple enumeration, analogy may suggest hypothesis to scientist.

(6) Relation between scientific thinking and logical thinking is one sided.

(7) The language which computer understand is called. 'artificial language'

(8) Logic gates are the basic building blocks of computer.

Q. 3. Explain the following.

(1) Application of logic in law.

(2) Application of logic in computer science.

(3) Role of logic in communication.

(4) Importance of logic in everyday life.

Q. 4. Answer the following questions.

(1) Explain with illustration how logic is useful in decision making.

(2) Explain with illustration application of logic in science.

(3) Explain role of logic in making and execution of laws.

(4) Explain how logic helps us to critically evaluate arguments.

❖ ❖ ❖

Glossary

Analogy : a form of induction involving inference from known resemblances to further resemblances.

Argument : a group of propositions in which one proposition is accepted on the evidence of the remaining ones.

Argumentum ad baculum : the non-formal fallacy in which there is appeal to force.

Argumentum ad hominem : the non-formal fallacy which involves personal attack.

Argumentum ad ignoratiam : the non-formal fallacy in which a statement is taken to be proved, because its opposite cannot be disproved.

Argumentum ad misericordiam : the non-formal fallacy in which there is appeal to pity.

Argumentum ad populum : the non-formal fallacy in which there is appeal to emotions.

Argumentum ad verecundiam : the non-formal fallacy which involves appeal to improper authority.

Binary connective (operator) : a propositional connective which connects two propositions.

Complement of a class : the class of all objects that do not belong to it,

Compound proposition : a proposition which contains another proposition (or propositions) as a component.

Conclusion : in an argument,the statement which is derived from the premises.

Conjunctive proposition : a compound proposition formed by combining any two propositions with the truth-functional connective "and".

Conjunctive truth function : truth-function which is true only when both the components are true.

Contingency : a truth-functional form which is true under some truth possibilities of its components, and false under other truth possibilities.

Contradiction : a truth-functional propositional form which is false under all truth possibilities of its components.

Contradictory function : another name for negation, its truth-value being the opposite of the truth value of the component proposition.

Converse fallacy of accident : the non-formal fallacy in which we point to a special case to assert a general statement.

Decision procedure : a method for deciding whether an object belongs to a certain class.

Deductive proof : a proof of the validity of an argument in which the conclusion is deducted from the premises by a sequence of (valid) elementary arguments.

Deductive argument : an argument in which the premises claim to provide sufficient evidence for the conclusion.

Direct deductive proof : the deductive proof in which the conclusion is deduced from the premises, by a sequence of (valid) elementary arguments.

Disjunctive proposition : a compound proposition in which the word "or" combines two propositions.

Disjunctive function : the truth function which is false only if both the components are false.

Dyadic connective (Operator) : a propositional connective which connects two propositions.

Equivalence : the propositional connective which is true when both its components have the same truth value.

Equivalent proposition : a compound proposition in which two component propositions materially imply each other.

Fallacy : an error in reasoning in which the argument appears to establish a conclusion, but does not really do so.

Fallacy of Accident : a non-formal fallacy in which what is true in general is considered to be true in a special case,or what is true under normal circumstances is taken to be true under special (or exceptional) circumstances.

Fallacy of Composition : a non-formal fallacy in which it is argued that a quality which is possessed by a member (or members) is also possessed by the group, or that quality which is possessed by a part (or parts) is also possessed by the whole.

Fallacy of Division : a non-formal fallacy in which it is argued that what is true of a group is true of its members or that what is true of a whole is true of its parts.

Fallacy of ignoratio elenchi : a group of non-formal fallacies in which the argument is irrelevant.

Formal fallacy : a fallacy which arises due to the violation of a rule of logic.

Implicative function : the truth function which is false if and only if the antecedent is true and the consequent is false.

Implicative proposition : a compound propositions which is formed by combining any two propositions with the truth-functional connective "if.. then..."

Inference : the process of reasoning in which the conclusion is drawn from the evidence.

Inductive arguments : an argument in which the premises provide "some" evidence for the conclusion, but the evidence is not sufficient.

Induction per simple enumeration : a generalization in which it is argued that what is true of several instances of a kind is true universally of that kind.

Monadic connective (operator) : a proposition connective which operates on one proposition.

Negation : the propositional connective "~".

Negative proposition : a compound proposition obtained by denying a proposition.

Non-formal fallacy : a fallacy which arises either when words are used ambiguously or when some relevant feature of the argument is ignored.

Premise : in an argument, the proposition from which the conclusion is drawn.

Proposition : a statement which is either true or false

Propositional connective : an expression which connects propositions. The symbols for the five propositional connectives are "~", ".", "v", "⊃" and "≡".

Propositional constant : a symbol which stands for a specific proposition.

Propositional variable : a symbol which stands for any proposition whatsoever.

Scientific induction : the process of establishing a general statement which is supported by both direct and indirect evidence.

Simple proposition : a proposition which does not contain any other proposition as a component.

Sound argument : a valid argument whose conclusion is a true proposition.

Tautology : a truth-functional propositional form which is true under all truth possibilities of its components.

Truth-functional connective (operator) : another name for propositional connective.

Truth-functionally compound proposition : a compound proposition whose truth value is determined by the truth value of its component proposition (or propositions).

Truth-table : a tabular way of expressing the truth values of expressions containing propositional connective.

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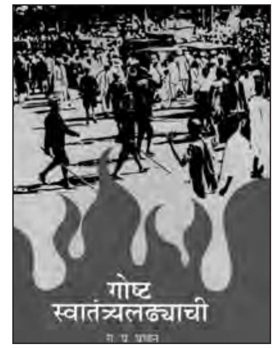
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भाग २
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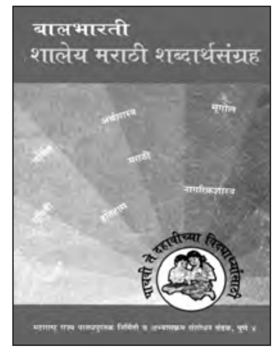
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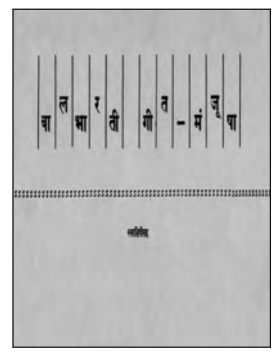
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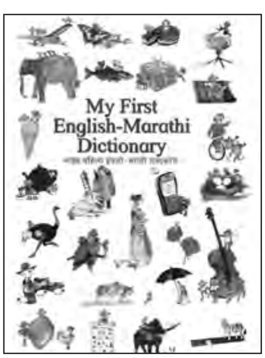
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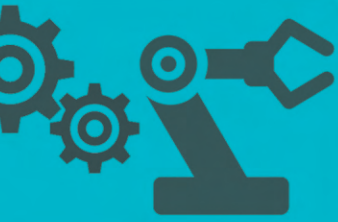

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